COPPALINI ARCHITECTURE

Suture Field

Solder flux and metric g

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Abstract

Spacetime interval and oriented surface flux arise from one rank-four field. The field sutures the tangent carrier of a Lorentzian manifold into an eight-address internal carrier. Its symmetric contraction is the metric g; its antisymmetric contraction is the solder flux $\mathcal{F}_{\mathrm{sol}}$. The sevenfold address action preserves both contractions, while the mirror preserves the metric and reverses flux orientation. Einstein curvature and Maxwell curvature meet when the induced flux satisfies the Maxwell field equations under a declared dimensional calibration.

1. Suture relation

Let M be a smooth four-manifold. Let $E \to M$ be a real rank-eight vector bundle whose fibers carry a symmetric form $\widehat{\eta}$, an antisymmetric form $\widehat{C}$, a cyclic action $\widehat{\varrho}(\mathbb{Z}_7)$, and a mirror $\widehat{H}$. A suture field is a smooth E-valued one-form

$$\Pi = \Pi^{A}{}_{\mu}\, dx^{\mu} \otimes e_A \in \Omega^1(M, E)$$

of constant rank four such that

$$g_{\mu\nu} := \widehat{\eta}_{AB}\Pi^{A}{}_{\mu}\Pi^{B}{}_{\nu} \quad \text{has signature } (3,1). \tag{SF.0}$$

Equivalently, at every address $x \in M$,

$$\Pi_x : T_xM \hookrightarrow E_x, \qquad g_x(u,v) = \widehat{\eta}_x(\Pi_x u, \Pi_x v). \tag{SF.1}$$

Put $L_\Pi := \operatorname{im}\Pi \subset E$. The rank and signature conditions make L_Π a Lorentzian rank-four subbundle and give a fiberwise isometry

$$\Pi : TM \xrightarrow{\cong} L_\Pi. \tag{SF.2}$$

Thus Π is an extended solder coframe. The word *suture* names the same coframe together with its four internal transverse address directions.

2. Eight-address carrier

Let

$$\mathcal{A}_8 := \{\infty\} \sqcup \mathbb{Z}_7, \qquad (\infty, 0, 1, 2, 3, 4, 5, 6) = (\mathsf{Gr}, \mathsf{Li}, \mathsf{Me}, \mathsf{Ti}, \mathsf{Hz}, \mathsf{Si}, \mathsf{Sc}, \mathsf{Mo}).$$

For $r \in \mathbb{Z}_7$, set

$$\mathsf{M}_r = \{\{\infty, r\}\} \cup \{\{r+j, r-j\} : j = 1, 2, 3\}.$$

Then $E(K_8) = \bigsqcup_{r\in\mathbb{Z}_7} \mathsf{M}_r$, and the canonical matching is

$$\mathsf{M}_0 = \{\mathsf{Gravity}-\mathsf{Light},\ \mathsf{Memory}-\mathsf{Motion},\ \mathsf{Time}-\mathsf{Scope},\ \mathsf{Horizon}-\mathsf{Silence}\}.$$

Let

$$\mathcal{V}_8 := \mathbb{R}[\mathcal{A}_8], \qquad \tau e_\infty = e_\infty, \quad \tau e_k = e_{k+1}. \tag{SF.3}$$

The shift permutes the matchings: $\tau\mathsf{M}_r = \mathsf{M}_{r+1}$. The cyclic action is read in the real Fourier decomposition of $\mathcal{V}_8$. Let

$$u_0 = \frac{1}{\sqrt{7}} \sum_{k=0}^{6} e_k,$$

and, for $j = 1, 2, 3$, let

$$\begin{aligned} c_j &= \sqrt{\frac{2}{7}} \sum_{k=0}^{6} \cos\!\left(\frac{2\pi jk}{7}\right) e_k, \\ s_j &= \sqrt{\frac{2}{7}} \sum_{k=0}^{6} \sin\!\left(\frac{2\pi jk}{7}\right) e_k. \end{aligned} \tag{SF.4}$$

Then

$$\mathcal{V}_8 = \operatorname{span}(e_\infty, u_0) \oplus \bigoplus_{j=1}^{3} \operatorname{span}(c_j, s_j). \tag{SF.5}$$

Writing $U_\alpha = e^{\alpha C}$, the address action becomes

$$\widehat{\varrho}(\tau^r) = I_2 \oplus U_{2\pi r/7} \oplus U_{4\pi r/7} \oplus U_{6\pi r/7}, \qquad r \in \mathbb{Z}_7. \tag{SF.6}$$

The first I_2 acts on $\operatorname{span}(e_\infty, u_0)$; the three rotations act on the ordered pairs (c_j, s_j). Hence SF.6 is a real eight-dimensional realization of the vertex permutation action in SF.3.

Let

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad C = -\Omega,$$

$$G = [Q, \Omega] = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}, \qquad H = \frac{G}{\sqrt{5}}, \qquad S = \frac{2Q - I}{\sqrt{5}}.$$

The common exact identities are

$$Q^2 = Q + I, \qquad C^2 = -I, \qquad G^2 = 5I, \qquad H^2 = S^2 = I,$$

$$H = CS, \qquad SG = \sqrt{5}\Omega = -GS, \qquad HC = -CH, \qquad HCH = -C,$$

$$L_s := e^{sS}, \qquad L_s^{\mathsf{T}} G L_s = G, \qquad H L_s H = L_{-s}.$$

On the Fourier carrier, define

$$\widehat{\eta} := G \oplus I_2 \oplus I_2 \oplus I_2, \qquad \widehat{C} := C \oplus C \oplus C \oplus C, \tag{SF.7}$$

and, with $D = \operatorname{diag}(1, -1)$,

$$\widehat{H} := H \oplus D \oplus D \oplus D. \tag{SF.8}$$

The symmetric form has signature $(7, 1)$. The first summand $\operatorname{span}(e_\infty, u_0)$ is the Gr–$\overline{\mathsf{U}}$ carrier: gravity $e_\infty = \mathsf{Gr}$ and the cycle mean $\overline{\mathsf{U}} := u_0$. The address $e_0 = \mathsf{Li}$ has weight $1/7$ on u_0 and $6/7$ on the transverse planes, so Li is not a summand of its own. The remaining three planes carry the nontrivial real Fourier weights of the sevenfold address cycle.

Theorem 1 (Cyclic and mirror invariance)**.** *For every $r \in \mathbb{Z}_7$,*

$$\widehat{\varrho}(\tau)^7 = I_8, \qquad \widehat{\varrho}(\tau^r)^{\mathsf{T}} \widehat{\eta} \widehat{\varrho}(\tau^r) = \widehat{\eta}, \qquad \widehat{\varrho}(\tau^r)^{\mathsf{T}} \widehat{C} \widehat{\varrho}(\tau^r) = \widehat{C}, \tag{SF.9}$$

and

$$\widehat{H}^{\mathsf{T}} \widehat{\eta} \widehat{H} = \widehat{\eta}, \qquad \widehat{H}^{\mathsf{T}} \widehat{C} \widehat{H} = -\widehat{C}, \qquad \widehat{H} \widehat{\varrho}(\tau^r) \widehat{H} = \widehat{\varrho}(\tau^{-r}). \tag{SF.10}$$

Proof. The three transverse blocks have eigenvalues $e^{\pm 2\pi i r/7}$, $e^{\pm 4\pi i r/7}$, and $e^{\pm 6\pi i r/7}$, while the root block is fixed. Thus seven cyclic steps give I_8. Every U_α preserves both I_2 and C. The shared root gives $H^{\mathsf{T}} G H = G$ and $HCH = -C$; each D gives $D^{\mathsf{T}} I_2 D = I_2$ and $DCD = -C$. Finally, $DU_\alpha D = U_{-\alpha}$. Taking direct sums gives SF.9–SF.10. □

3. Metric g

For a locally constant $r : U \to \mathbb{Z}_7$, the internal cyclic update is

$$\Pi \longmapsto \Pi^{(r)} := \widehat{\varrho}(\tau^r)\Pi. \tag{SF.11}$$

The metric is invariant:

$$\begin{aligned} g^{(r)}_{\mu\nu} &= \widehat{\eta}_{AB} (\Pi^{(r)})^A{}_\mu (\Pi^{(r)})^B{}_\nu \\ &= \widehat{\eta}_{AB} \Pi^A{}_\mu \Pi^B{}_\nu = g_{\mu\nu}. \end{aligned} \tag{SF.12}$$

The mirror does not commute with the cyclic action. By SF.10, $\widehat{H}\widehat{\varrho}(\tau^w) = \widehat{\varrho}(\tau^{-w})\widehat{H}$, so $\widehat{H}$ intertwines $\widehat{\varrho}$ with $\widehat{\varrho} \circ \sigma$. In the gauged case of §6 (principal $\mathbb{Z}_7$-bundle $P \to M$, $E = P \times_{\widehat{\varrho}} \mathcal{V}_8$), set

$$\sigma(\tau^w) = \tau^{-w}, \qquad E^\sigma := P \times_{\widehat{\varrho}\circ\sigma} \mathcal{V}_8, \qquad \bar{\Pi} := \widehat{H}\Pi \in \Omega^1(M, E^\sigma), \qquad \bar{\Pi}^{(w)} = \widehat{\varrho}(\tau^{-w})\bar{\Pi} = \widehat{H}\Pi^{(w)}. \tag{SF.13}$$

Both bundles carry the same $\widehat{\eta}$ and $\widehat{C}$, and $\widehat{H} : (E, \widehat{\eta}, \widehat{C}) \to (E^\sigma, \widehat{\eta}, -\widehat{C})$ is an isometry that reverses $\widehat{C}$. In particular

$$g(\bar{\Pi}) = g(\Pi). \tag{SF.13a}$$

The Levi–Civita connection and curvature are therefore determined by the suture metric:

$$\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}), \qquad \mathrm{Ein}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{SF.14}$$

4. Solder flux

The antisymmetric contraction of the same field is the solder flux

$$\mathcal{F}_{\mathrm{sol}} := \frac{1}{2} \widehat{C}_{AB} \Pi^A \wedge \Pi^B \in \Omega^2(M). \tag{SF.15}$$

For every oriented surface $\Sigma \subset M$, its integrated flux is

$$\Phi_{\mathrm{sol}}(\Sigma) := \int_\Sigma \mathcal{F}_{\mathrm{sol}}. \tag{SF.16}$$

The same cyclic update that preserves g preserves $\mathcal{F}_{\mathrm{sol}}$, while the mirror reverses its orientation:

$$\mathcal{F}_{\mathrm{sol}}(\Pi^{(r)}) = \mathcal{F}_{\mathrm{sol}}(\Pi), \qquad \mathcal{F}_{\mathrm{sol}}(\bar{\Pi}) = -\mathcal{F}_{\mathrm{sol}}(\Pi). \tag{SF.17}$$

Metric and flux thus arise at the same address from the symmetric and antisymmetric contractions of Π.

5. Electromagnetic and gravitational realization

Let χ be a declared constant dimensional calibration. An electromagnetic realization of solder flux is the field

$$F_{\mathrm{em}} := \chi \mathcal{F}_{\mathrm{sol}} \tag{SF.18}$$

when it satisfies

$$dF_{\mathrm{em}} = 0, \qquad d \star_g F_{\mathrm{em}} = \mu_0 \star_g J_{\mathrm{em}}. \tag{SF.19}$$

On a contractible region, the first condition gives a potential A with $F_{\mathrm{em}} = dA$. Its $U(1)$ loop read is

$$W_\gamma^{U(1)} = \exp\!\left(\frac{iq}{\hbar}\oint_\gamma A\right) = \exp\!\left(\frac{iq}{\hbar}\int_\Sigma F_{\mathrm{em}}\right) \quad (\partial\Sigma = \gamma). \tag{SF.20}$$

The electromagnetic stress–energy tensor is

$$T^{\mathrm{em}}_{\mu\nu} = \frac{1}{\mu_0}\left(F_{\mathrm{em}\,\mu\alpha}F_{\mathrm{em}\,\nu}{}^{\alpha} - \frac{1}{4}g_{\mu\nu}F_{\mathrm{em}\,\alpha\beta}F^{\alpha\beta}_{\mathrm{em}}\right). \tag{SF.21}$$

With material stress–energy $T^{\mathrm{mat}}_{\mu\nu}$, curvature obeys

$$\mathrm{Ein}_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4}\left(T^{\mathrm{mat}}_{\mu\nu} + T^{\mathrm{em}}_{\mu\nu}\right). \tag{SF.22}$$

Thus the suture metric supplies interval and curvature, and the calibrated solder flux supplies the electromagnetic two-form and its energy–momentum.

6. Sevenfold holonomy

Let $P \to M$ be a principal $\mathbb{Z}_7$ bundle with associated internal bundle $E = P \times_{\widehat{\varrho}} \mathcal{V}_8$. If a closed loop γ has monodromy τ^{w_γ}, $w_\gamma \in \mathbb{Z}_7$, its internal holonomy is

$$W_\gamma^{(7)} = \widehat{\varrho}(\tau^{w_\gamma}). \tag{SF.23}$$

The mirror field $\bar{\Pi}$ of SF.13 is a section of E^σ; its holonomy around γ is $\widehat{\varrho}(\tau^{-w_\gamma})$. The three nontrivial Fourier planes read its phase as

$$\exp\!\left(\frac{2\pi i j w_\gamma}{7}\right), \qquad j = 1, 2, 3. \tag{SF.24}$$

The $\mathbb{Z}_7$ holonomy and the $U(1)$ holonomy have distinct carriers. They meet in one physical realization when a material coupling and a calibrated observable relate $W_\gamma^{(7)}$ to F_{em} at the same address.

7. Direct occurrence

A physical field has an interface

$$\begin{aligned}\mathfrak{I}_J := (&J, \Gamma_J, \mathrm{Src}_J; A_J, X_J, Y_J, T_J, \Sigma_J, \mathrm{Con}_J,\\ &\rho_J, O_J, K_J, B_J, \theta_J), \qquad \rho_J : X_J \to Y_J, \quad O_J : X_J \to \mathcal{O}_J, \quad \theta_J : X_J \to \{0,1\}.\end{aligned} \tag{SF.25}$$

Its occurrence type is

$$\begin{aligned}\mathsf{Occ}_J := \{&(\mathfrak{I}_J; \Theta, x, \rho_J(x), \kappa, \boldsymbol{\varepsilon}, \theta_J(x)) :\\ &\Theta \in A_J,\ x \in X_J,\ \kappa \in K_J,\ \boldsymbol{\varepsilon} \in \mathbb{R}^3_{\geq 0}\}.\end{aligned} \tag{SF.26}$$

An indexed occurrence $\mathsf{Occ}_{J,n}$ is an element of Occ_J.

For $J = \mathsf{SF}$, take the state coordinates to include

$$x_n = (\Pi_n, g_n, (\mathcal{F}_{\mathrm{sol}})_n, F_{\mathrm{em},n}, J_{\mathrm{em},n}, W^{(7)}_{\gamma,n}), \tag{SF.27}$$

whenever the Maxwell conditions in SF.19 have been supplied. The suture projection is

$$\mathrm{pr}_{\mathrm{SF}}(\mathsf{Occ}_{\mathsf{SF},n}) = (\Theta_n, g_n, (\mathcal{F}_{\mathrm{sol}})_n, F_{\mathrm{em},n}, J_{\mathrm{em},n}, W^{(7)}_{\gamma,n}, O_n, \kappa_n, \boldsymbol{\varepsilon}_n). \tag{SF.28}$$

The metric, flux, calibration, current, loop read, observable, and residual remain separately measured coordinates of one occurrence.

8. Suture closure

At every spacetime address, the field closure is

$$\boxed{\Pi \in \Omega^1(M, E) \quad \longmapsto \quad \left(g = \Pi^* \widehat{\eta},\ \mathcal{F}_{\mathrm{sol}} = \Pi^* \widehat{C}\right).} \tag{SF.29}$$

Here the first pullback is symmetric and the second is antisymmetric:

$$\begin{aligned} \text{Suture Field } \Pi &: TM \xrightarrow{\cong} L_\Pi \subset E, \\ \text{Metric } g &: \text{interval, connection, and curvature,} \\ \text{Solder Flux } \mathcal{F}_{\mathrm{sol}} &: \text{oriented surface relation and loop phase.} \end{aligned} \tag{SF.30}$$

The cyclic address action preserves both g and $\mathcal{F}_{\mathrm{sol}}$; the mirror preserves g and reverses $\mathcal{F}_{\mathrm{sol}}$. The Maxwell conditions select the electromagnetic realization of flux, and the Einstein equation couples its stress–energy to spacetime curvature.