COPPALINI ARCHITECTURE

Suture Field Dynamics

Regular Hamiltonian reduction

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9 November 2026

Abstract

The suture field supplies an eight-dimensional internal carrier for a Lorentzian metric and an antisymmetric flux form. This note gives the exact condition under which its multiplier action has the same propagating content as Einstein–Maxwell theory. The condition is the rank-sixteen regularity of the pointwise suture map from the thirty-two coframe coordinates to the ten metric and six flux coordinates. On that branch the remaining sixteen coframe coordinates are vertical gauge coordinates; the multiplier sector is second-class; and the reduced Hamiltonian carries two gravitational and two electromagnetic polarizations. The reduced theory is Einstein–Maxwell on the domain where the rest-frame magnetic invariant of the flux is below the calibration, $b < |\chi|$; the electric invariant is unbounded.

1. Constrained action

Let $\Pi^A{}_\mu$ be a rank-four E-valued coframe, with $A = 1, \dots, 8$. Let $B^{\mu\nu} = -B^{\nu\mu}$ and $\lambda^{\mu\nu} = \lambda^{\nu\mu}$. Fix a nonzero constant calibration χ. Define

$$h_{\mu\nu}(\Pi) := \widehat{\eta}_{AB}\Pi^A{}_\mu\Pi^B{}_\nu, \qquad f_{\mu\nu}(\Pi) := \chi\widehat{C}_{AB}\Pi^A{}_\mu\Pi^B{}_\nu. \tag{SD.0}$$

The action is

$$\begin{aligned}\mathcal{S} = \int_M \Bigg[&\frac{c^3}{16\pi G_N}(R(g) - 2\Lambda) - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{1}{2}B^{\mu\nu}\big(F_{\mu\nu} - f_{\mu\nu}(\Pi)\big) + \lambda^{\mu\nu}\big(g_{\mu\nu} - h_{\mu\nu}(\Pi)\big)\Bigg]\sqrt{-g}\,d^4x, \qquad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}.\end{aligned} \tag{SD.1}$$

Variation of the multipliers gives the two suture conditions

$$F_{\mu\nu} = f_{\mu\nu}(\Pi), \qquad g_{\mu\nu} = h_{\mu\nu}(\Pi). \tag{SD.2}$$

They are constraint equations. Their multiplier momenta belong to the canonical system and remain present until Dirac reduction. In particular, variation with respect to B and λ gives SD.2; it does not eliminate B or *lambda*. Their values can only be related to the other equations after the coupled Euler system has been imposed, and that later relation does not remove their primary momenta from the Dirac analysis.

2. The pointwise suture map

At one spacetime address, define

$$\mathscr{S} : \mathrm{Hom}(\mathbb{R}^4, \mathbb{R}^8)_{\mathrm{rank}\,4} \longrightarrow \mathrm{Sym}^2(\mathbb{R}^4)^* \oplus \Lambda^2(\mathbb{R}^4)^*, \qquad \Pi \longmapsto (h(\Pi), f(\Pi)). \tag{SD.3}$$

The domain has dimension 32 and the target has dimension $10 + 6 = 16$. Its differential is

$$\begin{aligned}(D\mathscr{S}_\Pi\,\delta\Pi)^{(g)}_{\mu\nu} &= \widehat{\eta}_{AB}\big(\delta\Pi^A{}_\mu\Pi^B{}_\nu + \Pi^A{}_\mu\delta\Pi^B{}_\nu\big),\\ (D\mathscr{S}_\Pi\,\delta\Pi)^{(F)}_{\mu\nu} &= \chi\widehat{C}_{AB}\big(\delta\Pi^A{}_\mu\Pi^B{}_\nu + \Pi^A{}_\mu\delta\Pi^B{}_\nu\big).\end{aligned} \tag{SD.4}$$

Definition 1 (Regular suture branch)**.** For the fixed $\chi \neq 0$, the regular branch is the open locus on which

$$\operatorname{rank} D\mathscr{S}_\Pi = 16. \tag{SD.5}$$

An exact certificate that this locus is nonempty is

$$\Pi_\star = \begin{pmatrix} -1 & -2 & 1 & 0\\ 2 & -1 & 1 & 0\\ -1 & -2 & 1 & -2\\ -1 & 1 & -1 & 0\\ -1 & 0 & 0 & -1\\ 0 & 1 & 1 & -1\\ 2 & 2 & 1 & -2\\ -1 & -1 & 0 & -1 \end{pmatrix}. \tag{SD.6}$$

For $\chi = 1$, exact row reduction gives

$$\operatorname{rank} D\mathscr{S}_{\Pi_\star} = 16, \qquad h(\Pi_\star) = \begin{pmatrix} 10 & -5 & 9 & 0\\ -5 & 9 & -1 & 0\\ 9 & -1 & 6 & -5\\ 0 & 0 & -5 & 11 \end{pmatrix}. \tag{SD.7}$$

Its leading principal minors are

$$10, \qquad 65, \qquad -259, \qquad -4474, \tag{SD.8}$$

so the LDL^{T} pivot signs are $+, +, -, +$ and $h(\Pi_\star)$ has Lorentzian signature $(3, 1)$. The determinant of a nonzero 16×16 minor of $D\mathscr{S}$ remains nonzero on a neighborhood of $\Pi_\star$, which establishes the stated open regular branch.

3. Internal vertical directions

The linear internal stabilizer is

$$\mathfrak{k} = \Big\{X \in \mathfrak{gl}(8,\mathbb{R}) : X^{\mathsf{T}}\widehat{\eta} + \widehat{\eta}X = 0, \quad X^{\mathsf{T}}\widehat{C} + \widehat{C}X = 0\Big\}. \tag{SD.9}$$

Exact linear reduction gives

$$\dim\mathfrak{k} = 10, \qquad \mathfrak{k} \cong \mathfrak{so}(1,1) \oplus \mathfrak{u}(3). \tag{SD.10}$$

The ten linear directions $\delta\Pi = X\Pi$ preserve both h and f. The map $X \mapsto X\Pi$ is injective at $\Pi_\star$ and at Π_{vac} (SD.14a), exact rank 10, so $\mathfrak{k}$ acts freely there. On the regular branch,

$$\dim\ker D\mathscr{S}_\Pi = 32 - 16 = 16. \tag{SD.11}$$

The other six vertical directions are field-dependent coordinates along the same local fiber of $\mathscr{S}$. They carry no metric or flux coordinate.

4. Image of the suture map

For a Lorentzian h and $f \in \Lambda^2(\mathbb{R}^4)^*$, the endomorphism $h^{-1}f$ is h-antisymmetric, so its spectrum is $\{\pm ib, \pm e\}$ with $b, e \geq 0$:

$$b(h,f),\ e(h,f) \text{ are the rest-frame magnetic and electric invariants of } f. \tag{SD.11a}$$

For $f = 0$ and for null $f \neq 0$, $b = e = 0$.

Proposition 2 (Magnetic cap)**.** *Let* Π *have rank four with* $h = h(\Pi)$ *Lorentzian,* $f = f(\Pi)$*, and* $\chi \neq 0$*. Then*

$$(\chi^2 - b^2)(\chi^2 + e^2) = \chi^4 \, \frac{\det\big(h + \frac{i}{\chi} f\big)}{\det h} \;\geq\; 0, \qquad b \leq |\chi|. \tag{SD.11b}$$

Equality $b = |\chi|$ *holds only at critical points of* $\mathscr{S}$*; on the regular branch* $b < |\chi|$*.*

Proof. Put $F = \chi^{-1} f = \Pi^{\mathsf{T}} \widehat{C} \Pi$ and $H_c := \widehat{\eta} + i\widehat{C}$, a Hermitian matrix. Then $h + iF = \Pi^{\dagger} H_c \Pi$. On a Euclidean block $I_2 + iC$ has eigenvalues $0, 2$; on the root block $\det(G + iC) = -6$, so it has signature $(1, 1)$. Hence H_c has signature $(4, 1)$ with a three-dimensional kernel, and its restriction to $\operatorname{im}\Pi \otimes \mathbb{C}$ has at most one negative eigenvalue. A vector $w \in \mathbb{R}^4$ with $h(w, w) < 0$ gives $(\Pi w)^{\dagger} H_c \, \Pi w = h(w, w) < 0$, because the $\widehat{C}$ term vanishes by antisymmetry. So the restriction has exactly one, and $\det(h + iF) \leq 0$; also $\det h < 0$. For the spectrum $\{\pm i b_F, \pm e_F\}$ of $h^{-1}F$,

$$(1 - b_F^2)(1 + e_F^2) = \det(iI - h^{-1}F) = \frac{\det(ih - F)}{\det h} = \frac{\det(h + iF)}{\det h} \;\geq\; 0,$$

since $ih - F = i(h + iF)$. With $b = |\chi| b_F$ and $e = |\chi| e_F$ this is SD.11b. Suppose $b = |\chi|$ at a regular Π. Then $\mathscr{S}$ is a submersion there, so $(h, (1 + \varepsilon) f)$ lies in its image for small ε, with magnetic invariant $(1 + \varepsilon)|\chi| > |\chi|$ for $\varepsilon > 0$, contradicting the cap. □

Write $\mathbf{o}_1, \ldots, \mathbf{o}_8$ for the basis in which $\widehat{\eta} = G \oplus I_2 \oplus I_2 \oplus I_2$ and $\widehat{C} = C \oplus C \oplus C \oplus C$: $\mathbf{o}_{1,2}$ span the root plane and $\mathbf{o}_{3,4}$, $\mathbf{o}_{5,6}$, $\mathbf{o}_{7,8}$ the transverse planes. Let π_μ denote the μ-th column of Π.

Proposition 3 (Regular family)**.** *For* $a \in \mathbb{R}$ *and* $c^2 + s^2 = 1$ *let*

$$\pi_0 = x\,\mathbf{o}_1 + a\,\mathbf{o}_7, \quad \pi_1 = \mathbf{o}_8, \quad \pi_2 = \mathbf{o}_3, \quad \pi_3 = c\,\mathbf{o}_4 + s\,\mathbf{o}_5, \qquad x = \sqrt{\tfrac{1+a^2}{2}}. \tag{SD.11c}$$

Then $h = \operatorname{diag}(-1, 1, 1, 1)$ *and* $f = -\chi\big(a\,\theta^0 \wedge \theta^1 + c\,\theta^2 \wedge \theta^3\big)$*, so* $b = |\chi c|$ *and* $e = |\chi a|$*. A* 16×16 *minor of* $D\mathscr{S}$ *on a fixed column set equals* $-16\sqrt{2}\,\chi^6 s^3 (1 + a^2)^{7/2}$*; the family is regular iff* $s \neq 0$*, that is, iff* $b < |\chi|$*.*

Proposition 4 (Null witness)**.** *With* $x_0 = \frac{\sqrt{6}}{2}$*,* $x_1 = \frac{\sqrt{6}(4-\sqrt{2})}{24}$*,* $y_1 = -\frac{\sqrt{3}}{6}$*,* $\beta = \frac{\sqrt{2}}{4}$*, let*

$$\pi_0 = x_0 \mathbf{o}_1 + \mathbf{o}_4 + \mathbf{o}_7, \quad \pi_1 = x_1 \mathbf{o}_1 + y_1 \mathbf{o}_2 + \mathbf{o}_4 + \beta \mathbf{o}_8, \quad \pi_2 = \mathbf{o}_3, \quad \pi_3 = \mathbf{o}_5. \tag{SD.11d}$$

Then $h = \eta_4$*,* $f = \chi(\theta^0 + \theta^1) \wedge \theta^2$ *(null, nonzero), and* $\operatorname{rank} D\mathscr{S}_\Pi = 16$ *exactly in* $\mathbb{Q}(\sqrt{2}, \sqrt{3})$*.*

Theorem 5 (Image)**.**

$$\mathscr{S}(\textit{regular branch}) = \{(h, f) : \; h \textit{ Lorentzian}, \; b(h, f) < |\chi|\}. \tag{SD.11e}$$

Proof. "$\subseteq$" is SD.11b. For "$\supseteq$": for $A \in GL(4)$, $\mathscr{S}(\Pi A) = (A^{\mathsf{T}} h A, A^{\mathsf{T}} f A)$, and $D\mathscr{S}_{\Pi A}$ has the rank of $D\mathscr{S}_\Pi$. By the standard normal forms of a 2-form relative to a Lorentzian metric, a non-null f is congruent to $(\eta_4, \; \beta_0 \theta^2 \wedge \theta^3 + \xi_0 \theta^0 \wedge \theta^1)$ with $|\beta_0| = b$ and $|\xi_0| = e$; a null $f \neq 0$ is congruent to $(\eta_4, \; k(\theta^0 + \theta^1) \wedge \theta^2)$, where a boost sets $|k| = |\chi|$ and a reflection its sign; and $f = 0$ is congruent to $(\eta_4, 0)$. SD.11c with $a = -\xi_0/\chi$, $c = -\beta_0/\chi$ and $s = \sqrt{1 - c^2} > 0$ covers the non-null case, SD.11d the null case, and Π_{vac} of SD.14a the case $f = 0$. □

Consequently every solution of SD.1 on the regular branch satisfies $b(g, F) < |\chi|$ pointwise, every Einstein–Maxwell configuration with $b(g, F) < |\chi|$ lies in the image, and e is unbounded.

Boundary 6. The normal forms used in SD.11e are standard and are cited, not proved. The value of χ and the units of b are fixed only by calibration: $|\chi|$ must exceed the largest b of the configurations the reduction is to cover. No empirical comparison is asserted.

5. The coframe bracket matrix

The primary coframe momenta are $P_A{}^\mu \approx 0$. Their stabilization contains

$$\Psi_A{}^\mu = \sqrt{-g}\left(2\lambda^{\mu\nu}\widehat{\eta}_{AB}\Pi^B{}_\nu + \chi B^{\mu\nu}\widehat{C}_{AB}\Pi^B{}_\nu\right) \approx 0. \tag{SD.12}$$

With the canonical convention $\{\Pi^A{}_\mu(x), P_B{}^\nu(y)\} = \delta^A_B\delta^\nu_\mu\delta^3(x-y)$, the cross-bracket is

$$\{P_A{}^\mu(x), \Psi_B{}^\nu(y)\} = -\Delta_{AB}{}^{\mu\nu}(x)\,\delta^3(x-y), \qquad \Delta_{AB}{}^{\mu\nu} = 2\sqrt{-g}\left(\lambda^{\mu\nu}\widehat{\eta}_{AB} + \frac{\chi}{2}B^{\mu\nu}\widehat{C}_{AB}\right). \tag{SD.13}$$

Changing the Poisson-bracket ordering reverses this displayed overall sign; the rank statement is unchanged. At the vacuum multiplier point

$$\lambda^{\mu\nu} = 0, \qquad B^{\mu\nu} = 0, \tag{SD.14}$$

one has $\Delta = 0$. Thus nondegeneracy of $\widehat{\eta}$ and $\widehat{C}$, and rank four of Π, do not establish invertibility of Δ. The 32 coframe primaries and 32 displayed stabilizations therefore do not form a globally invertible second-class block.

This point occurs on a Lorentzian suture vacuum. Let the four columns of $\Pi_{\rm vac}$ be the first coordinate vector of each 2×2 carrier block. Then

$$h(\Pi_{\rm vac}) = \mathrm{diag}(-2,1,1,1), \qquad f(\Pi_{\rm vac}) = 0. \tag{SD.14a}$$

For $\Lambda = 0$, the constant metric $g = h(\Pi_{\rm vac})$ and $A = 0$ satisfy the vacuum Einstein–Maxwell equations with the multiplier values in SD.14.

This is not an isolated point. By SD.12, $\Psi_A{}^\mu = \sqrt{-g}\,m_I\,\partial q^I/\partial\Pi^A{}_\mu$ with m as in SD.17, and $D\mathscr{S}_\Pi$ has full row rank on the regular branch. So $\Psi \approx 0$ forces $m = 0$, that is

$$\lambda^{\mu\nu} = B^{\mu\nu} = 0, \qquad \Delta = 0 \quad \text{on the whole regular constraint surface,} \tag{SD.14b}$$

and SD.15a gives $Y^{\mu\nu} = \mu_0^{-1}\mathcal{F}^{\mu\nu}$ there.

6. Regular multiplier reduction

For an unambiguous Legendre transform, introduce an independent antisymmetric field $\mathcal{F}_{\mu\nu}$ and an antisymmetric multiplier $Y^{\mu\nu}$:

$$\begin{aligned}\mathcal{S}_{\rm fo} = \int_M \Bigg[&\frac{c^3}{16\pi G_N}(R-2\Lambda) - \frac{1}{4\mu_0}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\\ &+ \frac{1}{2}Y^{\mu\nu}\big(\mathcal{F}_{\mu\nu} - 2\partial_{[\mu}A_{\nu]}\big) + \frac{1}{2}B^{\mu\nu}\big(\mathcal{F}_{\mu\nu} - f_{\mu\nu}(\Pi)\big)\\ &+ \lambda^{\mu\nu}\big(g_{\mu\nu} - h_{\mu\nu}(\Pi)\big)\Bigg]\sqrt{-g}\,d^4x.\end{aligned} \tag{SD.15}$$

Variation of Y returns $\mathcal{F} = dA$, so SD.15 is a first-order form of SD.1. Variation of $\mathcal{F}$ gives

$$Y^{\mu\nu} + B^{\mu\nu} = \frac{1}{\mu_0}\mathcal{F}^{\mu\nu}. \tag{SD.15a}$$

Thus B remains an auxiliary multiplier variable through the canonical reduction; SD.15a is not a license to remove its primary constraint before that reduction.

By the submersion theorem, local coordinates on the regular coframe carrier are

$$\Pi \longleftrightarrow (q^I, \xi^a), \qquad q^I = \mathscr{S}^I(\Pi), \qquad I, a = 1,\dots,16. \tag{SD.16}$$

In these coordinates, the multiplier sector has normal form

$$\mathcal{L}_{\rm mult} = \sqrt{-g}\,m_I\big(X^I - q^I\big), \qquad X = (g,\mathcal{F}), \qquad m_{(\mu\nu)} = (2-\delta_{\mu\nu})\lambda^{\mu\nu}, \quad m_{[\mu\nu]} = B^{\mu\nu}\ (\mu<\nu). \tag{SD.17}$$

Here I runs over the ten components $\mu \le \nu$ of the symmetric sector and the six components $\mu < \nu$ of the antisymmetric sector, so that $\sum_{\mu\nu} \lambda^{\mu\nu}(g-h)_{\mu\nu} + \frac{1}{2}\sum_{\mu\nu} B^{\mu\nu}(\mathcal{F}-f)_{\mu\nu} = \sum_I m_I(X^I - q^I)$. Its three Euler relations are

$$X^I - q^I = 0, \qquad m_I = 0, \qquad \frac{\delta \mathcal{L}_{\mathrm{mult}}}{\delta \xi^a} = 0. \tag{SD.18}$$

The ξ^a are vertical gauge coordinates. The remaining auxiliary constraints admit the local Darboux pairing

$$(P_{qI}, q^I - X^I), \qquad (P_{mI}, m_I), \tag{SD.19}$$

which gives 64 second-class constraints. The 16 constraints

$$P_{\xi a} \approx 0 \tag{SD.20}$$

are first class and generate vertical suture motions.

More explicitly, write

$$\mathcal{C}_I := q^I - X^I. \tag{SD.20a}$$

The auxiliary Hamiltonian has the local form

$$H_{\mathrm{aux}} = \int d^3x \left(m_I \mathcal{C}_I + u^I_m P_{mI} + u^I_q P_{qI} + u^a_\xi P_{\xi a} \right). \tag{SD.20b}$$

Its primary stabilizations are

$$\dot{P}_{mI} \approx 0 \Longrightarrow \mathcal{C}_I \approx 0, \qquad \dot{P}_{qI} \approx 0 \Longrightarrow m_I \approx 0, \qquad \dot{P}_{\xi a} \approx 0 \Longrightarrow 0. \tag{SD.20c}$$

With ordered constraint vector

$$\boldsymbol{\zeta} = (P_q, \mathcal{C}; P_m, m), \tag{SD.20d}$$

the second-class matrix is

$$\{\boldsymbol{\zeta}(x), \boldsymbol{\zeta}(y)\} = \begin{pmatrix} 0 & -I_{16} & 0 & 0 \\ I_{16} & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_{16} \\ 0 & 0 & I_{16} & 0 \end{pmatrix} \delta^3(x-y). \tag{SD.20e}$$

It is invertible. Stabilization of $\mathcal{C}_I$ and m_I fixes u^I_q and u^I_m; it creates no tertiary auxiliary constraint.

Lapse and shift. Because N and N^i are components of g, the primary constraints $\pi_a = (\pi_N, \pi_i)$ have nonzero brackets with the $g_{0\mu}$-components of $\mathcal{C}_I$: $\{\pi_a, \mathcal{C}_J\} = \partial X^J / \partial n^a$ with $n^a = (N, N^i)$. The combinations

$$\tilde{\pi}_a := \pi_a + \frac{\partial X^I}{\partial n^a} P_{qI} \tag{SD.20f}$$

commute with every $\mathcal{C}_J$ and every P_{qJ}. The bracket matrix of $(\pi_a, P_q, \mathcal{C}; P_m, m)$ therefore has rank 64 and a four-dimensional kernel spanned by the $\tilde{\pi}_a$, which replace π_N, π_i in SD.24, SD.27 and SD.29. Their first-class status with respect to $\mathcal{H}_\perp, \mathcal{H}_i$ is not re-derived here.

Charts. The pairing SD.19 is written for SD.15, where $\mathcal{F}$ is an independent field: phase-space dimension 148, with 26 first-class and 88 second-class constraints, and DoF $= \frac{1}{2}(148 - 2\cdot 26 - 88) = 4$. For the direct action SD.1 counted in SD.28, F_{0i} is a velocity, and the same reduction holds with F_{0i} expressed through the momentum of A_i and B^{0i}: primary $5 + 32 + 6 + 10 = 53$, secondary $5 + 6 + 10 + 16 = 37$, total $90 = 26 + 64$.

Theorem 7 (Regular suture reduction)**.** *On the regular branch, Dirac reduction of SD.15 is locally equivalent, on the domain $b(g,F) < |\chi|$ of SD.11e, to*

$$\mathcal{S}_{\mathrm{red}} = \int_M \left[\frac{c^3}{16\pi G_N}(R - 2\Lambda) - \frac{1}{4\mu_0} F_{\mu\nu}F^{\mu\nu} \right] \sqrt{-g}\, d^4x, \qquad F = dA. \tag{SD.21}$$

Proof. SD.18 removes q and m while SD.20 removes the vertical coordinates. Variation of Y returns $\mathcal{F} = dA$. Substitution leaves SD.21. □

7. ADM Hamiltonian and stabilization

Let

$$K_{ij} = \frac{1}{2N}\big(\dot\gamma_{ij} - D_iN_j - D_jN_i\big), \qquad \pi^{ij} = \frac{c^3\sqrt{\gamma}}{16\pi G_N}\big(K^{ij} - \gamma^{ij}K\big), \tag{SD.22}$$

and, after the multiplier reduction, let

$$E^i = -\frac{\sqrt{-g}}{\mu_0}F^{0i}. \tag{SD.23}$$

The reduced primary constraints are

$$\pi_N \approx 0, \qquad \pi_i \approx 0, \qquad E^0 \approx 0. \tag{SD.24}$$

The total Hamiltonian is

$$H_T = \int d^3x \left(N\mathcal{H}_\perp + N^i\mathcal{H}_i + A_0\mathcal{G} + u_N\pi_N + u^i\pi_i + u_0E^0\right), \tag{SD.25}$$

where

$$\begin{aligned}\mathcal{H}_\perp = {}& \frac{16\pi G_N}{c^3\sqrt{\gamma}}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2\right) - \frac{c^3\sqrt{\gamma}}{16\pi G_N}\big({}^{(3)}R - 2\Lambda\big)\\ &+ \frac{\mu_0}{2\sqrt{\gamma}}\gamma_{ij}E^iE^j + \frac{\sqrt{\gamma}}{4\mu_0}F_{ij}F^{ij},\\ \mathcal{H}_i = {}& -2\gamma_{ik}D_j\pi^{jk} + F_{ij}E^j, \qquad \mathcal{G} = -\partial_iE^i.\end{aligned} \tag{SD.26}$$

Preservation of SD.24 gives

$$\dot\pi_N \approx 0 \Longrightarrow \mathcal{H}_\perp \approx 0, \qquad \dot\pi_i \approx 0 \Longrightarrow \mathcal{H}_i \approx 0, \qquad \dot E^0 \approx 0 \Longrightarrow \mathcal{G} \approx 0. \tag{SD.27}$$

The scalar, momentum, and Gauss constraints close under the standard hypersurface-deformation and $U(1)$ gauge algebra. Their stabilizations fix only the corresponding Dirac multipliers and produce no tertiary physical constraint.

8. Degree count

On the regular Darboux chart of the direct action SD.1, the phase-space dimensions per spatial address are:

sector	phase-space dimension
(γ_{ij}, N, N^i)	20
A_μ	8
$\Pi^A{}_\mu$	64
$B^{\mu\nu}$	12
$\lambda^{\mu\nu}$	20
total	124

(SD.28)

There are ten Einstein–Maxwell first-class constraints and sixteen vertical suture first-class constraints. The regular multiplier sector supplies sixty four second-class constraints. Therefore

$$\mathrm{DoF} = \frac{1}{2}\big(124 - 2(10+16) - 64\big) = 4. \tag{SD.29}$$

Equivalently, after exact reduction the Einstein–Maxwell phase space has dimension 28 and ten first-class constraints:

$$\mathrm{DoF}_{\mathrm{red}} = \frac{1}{2}\big(28 - 2\cdot 10\big) = 4. \tag{SD.30}$$

The four propagating degrees of freedom are the two gravitational and two electromagnetic transverse polarizations.

9. Discrete carrier

The $\mathbb{Z}_7$ Fourier action of the suture carrier preserves $\widehat{\eta}$ and $\widehat{C}$. In SD.1 it is an algebraic fiber symmetry and contributes no new canonical pair. A local connection or a measured holonomy coupling enters only through an additional specified dynamical term and calibrated occurrence record.