

A MINIMUM AXIAL-RETURN ARCHITECTURE

LINEAR INFINITY

Return Without Erasure in a Minimum Two-Form Architecture

$$\zeta_{\text{ax}}(z) = \frac{\Phi - z}{z - \Psi}$$

$$\zeta_{\text{ax}}(L_{\text{ax}}z) = \Phi^{-2}\zeta_{\text{ax}}(z) \implies \Lambda(L_{\text{ax}}z) = \Lambda(z) + 2\log \Phi$$

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Declaration. This paper proves an exact axial-return theorem from two real 2×2 generators. Its central result is neither a survey nor a numerical analogy. A projective coordinate turns the root carriage into translation; the quotient returns while the lifted address continues; a harmonic period retains the traversal.

Physical scope. The same carriage has a positive projective read and a signed Lorentzian read. Standard physical data may then be supplied and tested in their native equations. Generated structure, derived adapter, supplied realization, native constraint, and empirical Reader remain distinct.

Claim wall. Nothing in the root derives an empirical constant, a laboratory state, a four-dimensional matter field, or the whole content of an established theory. What is exact is the common operational diagram and each explicitly displayed realization of it.

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Abstract

Let $V = \mathbb{R}^2$ carry the standard alternating form and compatible quarter-turn C_1 , and let Q be the Fibonacci transport. Their commutator $G = [C_1, Q]$ is an involutive signed form after normalization. A second normalized involution is not independently chosen: it is welded to the first by

$$H = \frac{G}{\sqrt{5}}, \quad S = HC_1 = \frac{2Q - I}{\sqrt{5}}, \quad \{H, S\} = 0.$$

The positive carriage

$$L_{\text{ax}} = \frac{Q + 2I}{\sqrt{5}} = e^{(\log \Phi)S}, \quad L_{\text{ax}}^2 = Q^2,$$

is simultaneously a Lorentz boost on the signed carrier, a hyperbolic transvection in the projective plane, and a deck step on an axial quotient. The golden Möbius chart

$$\xi_{\text{ax}}(z) = \frac{\Phi - z}{z - \Psi}$$

rectifies its projective action to dilation. The centered coordinate

$$\Lambda(z) = \log \frac{\Phi}{|\xi_{\text{ax}}(z)|}$$

then makes the action exactly linear:

$$\Lambda(L_{\text{ax}}z) = \Lambda(z) + T, \quad T = 2 \log \Phi, \quad \Lambda(i) = 0.$$

On the quotient, $[\Lambda + T] = [\Lambda]$; on the cover, $\Lambda + T \neq \Lambda$; and the descended harmonic form has core period T . This conjunction—quotient closure, continuing lifted address, and retained period—is called *Linear Infinity*. It gives a typed definition of RETURN without reset. A single axial coframe further produces a positive hyperbolic metric and a Lorentzian $\widehat{\text{AdS}}_2$ metric without identifying their topology or causal use. Exact Gravity and Bell realizations are supplied as native tests of this architecture, with their imported physical data stated at the point of use.

The quotient returns. The lift continues.

The traversal remains readable.

The Linear Infinity theorem

Theorem 1 (Linear Infinity) *Let*

$$C_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Phi = \frac{1 + \sqrt{5}}{2}, \quad \Psi = \frac{1 - \sqrt{5}}{2}.$$

Define

$$G = [C_1, Q], \quad H = G/\sqrt{5}, \quad S = (2Q - I)/\sqrt{5}, \quad L_{\text{ax}} = (Q + 2I)/\sqrt{5}.$$

Then $S = HC_1$, $H^2 = S^2 = I$, $HS = C_1$, $SH = -C_1$, *and*

$$L_{\text{ax}} = e^{(\log \Phi)S}, \quad L_{\text{ax}}^2 = Q^2.$$

On the upper half-plane, the projective point action of L_{ax} obeys

$$\zeta_{\text{ax}}(L_{\text{ax}}z) = \Phi^{-2}\zeta_{\text{ax}}(z), \quad \zeta_{\text{ax}}(z) = \frac{\Phi - z}{z - \Psi}.$$

Consequently the centered coordinate

$$\Lambda(z) = \log \frac{\Phi}{|\zeta_{\text{ax}}(z)|}$$

satisfies

$$\Lambda(i) = 0, \quad \Lambda(L_{\text{ax}}z) = \Lambda(z) + 2 \log \Phi.$$

For $T = 2 \log \Phi$, let

$$p : \mathbb{H}^+ \longrightarrow M_{\text{ax}} = \langle L_{\text{ax}} \rangle \backslash \mathbb{H}^+$$

be the quotient map and let $\bar{\alpha}_{\text{ret}}$ be defined by $p^*\bar{\alpha}_{\text{ret}} = d\Lambda$. Then

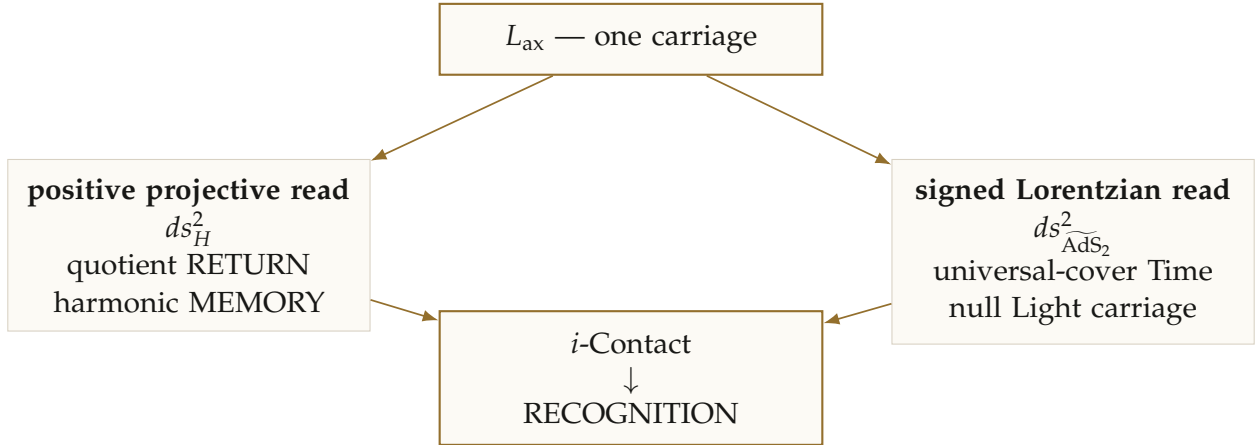
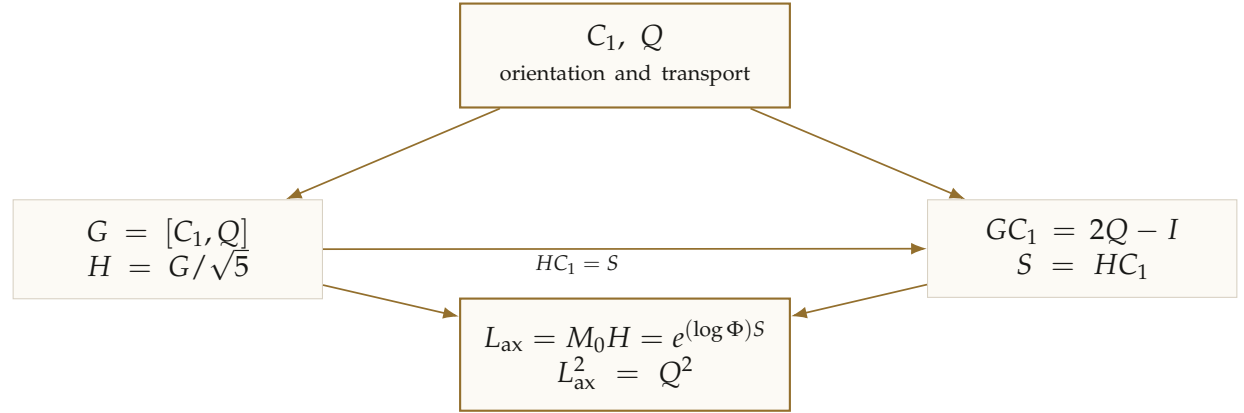
$$[\Lambda + T] = [\Lambda], \quad \Lambda + T \neq \Lambda \text{ on the cover}, \quad \oint_{\text{core}} \bar{\alpha}_{\text{ret}} = T.$$

Proof. Every matrix identity is established in Parts I and II. The displayed conjugacy sends the carriage to positive scalar multiplication by Φ^{-2} . Taking the centered negative logarithm of its modulus changes multiplication to addition. The quotient identifies translation by T , while the descended form retains that translation as its nonzero period. No identification of lifted points occurs. $\square$

Three infinities. The pole $\zeta_{\text{ax}}(\Psi) = \infty$, the unbounded cover coordinate $\Lambda \in \mathbb{R}$, and the infinite cyclic address $n \in \mathbb{Z}$ are connected but not identical. The chart ζ_{ax} is Möbius, not linear. It rectifies the carriage to dilation; the logarithmic coordinate linearizes that dilation to translation.

The double diamond

The proof has two compressions. The first constructs one carriage from a welded pair of root reads. The second keeps two geometries of that carriage separate until they meet at a typed Contact.



GENERATE $\neq$ REALIZE $\neq$ CONSTRAIN $\neq$ READ.

These offices compose. None impersonates another.

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THE ROOT

The minimum carrier

An oriented difference requires two places. The least real carrier of a nondegenerate alternating form is therefore

$$V = \mathbb{R}^2.$$

Let

$$C_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \Omega = -C_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The matrix Ω presents the alternating two-form

$$\omega(u, v) = u^T \Omega v.$$

It is nondegenerate, $\omega(u, u) = 0$, and changes sign when its arguments are exchanged. The vanishing of self-pairing is neutral contact; it does not erase the two-place relation. The compatible positive metric is

$$g_i(u, v) = \omega(u, C_1 v) = u^T v.$$

Thus the same carrier has orientation ω , quarter-turn C_1 , and a positive unit read g_i , without treating them as one object.

The wheel is exact:

$$C_1^2 = -I, \quad C_1^4 = I,$$

and no smaller positive power is I . Its order is not a display choice. It is a root output:

$$m = 4.$$

Namespace wall. The scalar $i_{\mathbb{C}}$, the real operator C_1 , and the projective point $i_{\mathbb{H}}$ are linked by explicit adapters but are not the same typed object. Likewise Ω is the displayed form matrix, while ω is the alternating bilinear map it presents.

Transport and complete planar algebra

The second primitive is the Fibonacci transport

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

It obeys

$$Q^2 = Q + I, \quad \det Q = -1, \quad \text{tr } Q = 1.$$

The matrices do not commute. In fact

$$I, \quad C_1, \quad Q, \quad C_1 Q$$

are linearly independent, so

$$\text{Alg}_{\mathbb{R}}(Q, C_1) = M_2(\mathbb{R}).$$

No planar linear motion lies outside the algebra generated by the root. This does not make every word a different reduced value. Word provenance and normal form remain separate:

$$\text{word } w(C_1, Q) \mapsto \text{matrix value } [w] \in M_2(\mathbb{R}).$$

Two words may reduce to one matrix and still record different executions. A receipt is a third object: evidence that a particular word was executed at a particular occurrence. The architecture keeps

$\text{address} \neq \text{provenance} \neq \text{receipt}.$

Difference and the welded mirrors

The commutator is

$$G = [C_1, Q] = C_1Q - QC_1 = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}.$$

It is symmetric, traceless, and satisfies

$$G^2 = 5I.$$

Normalize it:

$$H = \frac{G}{\sqrt{5}}.$$

Then $H^2 = I$, $H^T = H$, and its g_i -eigenvalues are $-1, +1$. It defines the signed unit form

$$\eta_G(u, v) = g_i(u, Hv) = u^T Hv$$

of signature $(1, 1)$. The subscript retains provenance: G is the root commutator, not the Einstein tensor or Newton's constant.

Now multiply the commutator by the root turn:

$$GC_1 = 2Q - I.$$

The normalized traceless transport is therefore not a new primitive:

$$S = \frac{2Q - I}{\sqrt{5}} = HC_1.$$

Direct multiplication closes the weld:

$$H^2 = S^2 = I, \quad HS = C_1, \quad SH = -C_1 = \Omega, \quad \{H, S\} = 0.$$

The Lorentz-generator identity is one line:

$$S^T G + GS = \sqrt{5}(SH + HS) = 0.$$

The two symmetric involutions are the same traceless content of Q read before and after the compatible quarter-turn. They are welded, not merely compatible.

$$S = HC_1.$$

The root Difference carried through the root Orientation is the boost generator of the positive carriage.

Golden boundary and null directions

The characteristic polynomial of Q is $\lambda^2 - \lambda - 1$. Its roots are

$$\Phi = \frac{1 + \sqrt{5}}{2}, \quad \Psi = \frac{1 - \sqrt{5}}{2},$$

with

$$\Phi + \Psi = 1, \quad \Phi\Psi = -1, \quad \Phi - \Psi = \sqrt{5}.$$

The eigenvectors

$$v_\Phi = \begin{pmatrix} \Phi \\ 1 \end{pmatrix}, \quad v_\Psi = \begin{pmatrix} \Psi \\ 1 \end{pmatrix}$$

are precisely the null directions of the signed form:

$$v_\Phi^\top G v_\Phi = 0, \quad v_\Psi^\top G v_\Psi = 0.$$

Their alternating separation is

$$\omega(v_\Phi, v_\Psi) = \sqrt{5}$$

up to the declared orientation. The golden numbers are not painted onto a later picture. They are the eigen-boundary of Q , the null boundary of G , and the ideal endpoints of the axial projective geodesic.

One-step reversal, two-step Return

Because $\det Q = -1$, one transport step reverses both root forms:

$$Q^\top \Omega Q = -\Omega, \quad Q^\top G Q = -G.$$

The next step restores them:

$$(Q^2)^\top \Omega Q^2 = \Omega, \quad (Q^2)^\top G Q^2 = G.$$

But

$$Q^2 = Q + I = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \neq I.$$

This is the first exact instance of the paper's central distinction: the form returns while the state advances. Return is not reset.

Define the root mirror

$$M_0 = Q\Omega.$$

Then $M_0^2 = I$. The orientation-preserving carriage is

$$L_{\text{ax}} = M_0 H = \frac{Q + 2I}{\sqrt{5}} = \frac{1}{\sqrt{5}} \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}.$$

It satisfies

$$\det L_{\text{ax}} = 1, \quad L_{\text{ax}}^2 = Q^2, \quad L_{\text{ax}}^T G L_{\text{ax}} = G.$$

Since $S^2 = I$, the golden identities

$$\cosh(\log \Phi) = \frac{\sqrt{5}}{2}, \quad \sinh(\log \Phi) = \frac{1}{2}$$

give

$$e^{(\log \Phi)S} = \cosh(\log \Phi)I + \sinh(\log \Phi)S = L_{\text{ax}}.$$

Thus L_{ax} is the positive square root of the two-step transport and the rapidity-log Φ boost generated by the welded involution S .

Representation wall. As a raw linear map,

$$L_{\text{ax}}^T L_{\text{ax}} = L_{\text{ax}}^2 = Q^2 \neq I.$$

It does not preserve the Euclidean form g_i . Its projective action preserves the Poincaré metric, and its linear action preserves η_G . Those are different representations of one element, not duplicate claims.

LINEAR INFINITY

The golden axis

Identify the projective action of a real 2×2 matrix with its Möbius action on the upper half-plane $\mathbb{H}^+$. The two eigen-directions of Q become the ideal boundary points Ψ and Φ . Their connecting geodesic is

$$\mathcal{A}_{\sqrt{5}} = \left\{ z = x + iy \in \mathbb{H}^+ : \left(x - \frac{1}{2} \right)^2 + y^2 = \frac{5}{4} \right\}.$$

The point i lies on this axis:

$$\left(0 - \frac{1}{2} \right)^2 + 1 = \frac{5}{4}.$$

The root carriage fixes the ideal endpoints and moves points along the axis. It acts by

$$z \mapsto L_{\text{ax}} z = \frac{3z + 1}{z + 2}$$

in projective coordinates. This curved presentation is exact but conceals the translation. The required rectifying chart is

$$\boxed{\tilde{\zeta}_{\text{ax}}(z) = \frac{\Phi - z}{z - \Psi}.$$

It sends

$$\tilde{\zeta}_{\text{ax}}(\Phi) = 0, \quad \tilde{\zeta}_{\text{ax}}(\Psi) = \infty,$$

and sends the golden axis to the imaginary axis. Its real Möbius matrix has determinant $-\sqrt{5}$, so it maps $\mathbb{H}^+$ to $\mathbb{H}^-$. This orientation fact is retained rather than silently repaired. Write

$$w = \tilde{\zeta}_{\text{ax}}(z) = X + iY, \quad Y < 0, \quad \hat{Y} = -Y > 0.$$

At the distinguished Contact point,

$$\boxed{\tilde{\zeta}_{\text{ax}}(i) = -i\Phi.}$$

Thus i is not introduced as a visual center. It is an exact point on the invariant axis at modulus Φ .

Rectification and the linear coordinate

The decisive conjugacy is

$$\boxed{\tilde{\zeta}_{\text{ax}}(L_{\text{ax}} z) = \Phi^{-2} \tilde{\zeta}_{\text{ax}}(z).}$$

The carriage is hyperbolic translation in its projective presentation and positive dilation in the rectified presentation. Define

$$\boxed{\Lambda(z) = \log \frac{\Phi}{|\tilde{\zeta}_{\text{ax}}(z)|}.$$

The numerator is a normalization, not a dynamical input. It centers the coordinate at the exact i -Contact:

$$\Lambda(i) = \log \frac{\Phi}{|-i\Phi|} = 0.$$

Now the action is literally additive:

$$\begin{aligned} \Lambda(L_{\text{ax}}z) &= \log \frac{\Phi}{|\Phi^{-2}\tilde{\zeta}_{\text{ax}}(z)|} \\ &= \log \frac{\Phi}{|\tilde{\zeta}_{\text{ax}}(z)|} + 2 \log \Phi \quad T := 2 \log \Phi. \\ &= \Lambda(z) + T, \end{aligned}$$

Iterating gives

$$\Lambda(L_{\text{ax}}^n z) = \Lambda(z) + nT, \quad n \in \mathbb{Z}.$$

This is the title in normal form. Infinity has not been compressed into a finite point, nor has curvature been denied. A lawful chart exposes the unbounded address as a straight translation.

curved axis $\longrightarrow$ dilation $\longrightarrow$ translation $\longrightarrow$ quotient Return | unbounded lift.

What is linear, and what is infinite

The Möbius map $\tilde{\zeta}_{\text{ax}}$ is not linear. It conjugates one projective action to a dilation. The logarithmic coordinate Λ linearizes that dilation. The term *Linear Infinity* names the resulting conjunction:

1. the lifted coordinate is unbounded in both directions;
2. the motion is a constant translation by T ;
3. the quotient position closes after one translation;
4. the lifted address n and harmonic period retain the traversal.

The ideal pole $\tilde{\zeta}_{\text{ax}}(\Psi) = \infty$, the real coordinate $\Lambda \in \mathbb{R}$, and the deck address $n \in \mathbb{Z}$ remain three typed objects.

Golden Fermi coordinates

Define the transverse coordinate

$$\delta = \operatorname{arsinh} \left(\frac{X}{\hat{Y}} \right).$$

Since $|w| = \Phi e^{-\Lambda}$, the inverse relations are

$$X = \Phi e^{-\Lambda} \tanh \delta, \quad \hat{Y} = \Phi e^{-\Lambda} \operatorname{sech} \delta.$$

The rail is the intrinsic axis

$$\delta = 0.$$

No Gaussian, aperture, or attraction law has been imposed. Substitution into the Poincaré metric gives

$$\frac{dX^2 + d\hat{Y}^2}{\hat{Y}^2} = d\delta^2 + \cosh^2 \delta d\Lambda^2.$$

Hence

$$\boxed{ds_H^2 = d\delta^2 + \cosh^2 \delta d\Lambda^2.}$$

The carriage preserves δ and advances Λ at a constant rate. The longitudinal coordinate vector ∂_Λ is Killing. Its norm is $\cosh \delta$. The unit longitudinal vector is

$$\operatorname{sech} \delta \partial_\Lambda,$$

whereas the metric dual of $d\Lambda$ is

$$(d\Lambda)^\sharp = \operatorname{sech}^2 \delta \partial_\Lambda.$$

These are not the same object. The native scalar profile

$$|d\Lambda| = \operatorname{sech} \delta$$

is a norm. Turning it into luminance, force, relaxation, or attraction would be an additional Reader or coupling, not a theorem of the geometry.

The quotient and the harmonic Memory class

Let

$$p : \mathbb{H}^+ \longrightarrow M_{\text{ax}} := \langle L_{\text{ax}} \rangle \backslash \mathbb{H}^+$$

be the quotient projection. Topologically M_{ax} is a cylinder, conformally an annulus. It is not a torus. In Fermi coordinates the quotient identifies

$$\Lambda \sim \Lambda + T.$$

The scalar Λ does not descend globally. Its differential does: there is a unique one-form $\bar{\alpha}_{\text{ret}}$ on the quotient such that

$$\boxed{p^* \bar{\alpha}_{\text{ret}} = d\Lambda.}$$

Choose the orientation $d\delta \wedge d\Lambda$. With the orthonormal coframe

$$e^1 = d\delta, \quad e^2 = \cosh \delta d\Lambda,$$

one has

$$*d\Lambda = -\operatorname{sech} \delta d\delta.$$

Therefore

$$d(d\Lambda) = 0, \quad d(*d\Lambda) = 0.$$

The descended form is harmonic and nonexact. Its period on the positively oriented core is

$$\oint_{\text{core}} \bar{a}_{\text{ret}} = T = 2 \log \Phi,$$

and its pointwise norm is

$$|\bar{a}_{\text{ret}}| = \text{sech } \delta.$$

On the cover, $d\Lambda$ is exact. On the quotient, the class

$$[\bar{a}_{\text{ret}}] \in H_{\text{dR}}^1(M_{\text{ax}})$$

records the nonzero core period. That topological distinction is the precise sense in which Memory survives quotient closure here. It invokes ordinary de Rham and Hodge theory on this explicit cylinder; it does not invoke the Hodge conjecture.

Two mirrors, one translation

Let

$$\mathcal{T}(z) = 1 - z, \quad K(z) = \bar{z}.$$

The golden chart gives the exact normal forms

$$\xi_{\text{ax}}(\mathcal{T}z) = \frac{1}{\xi_{\text{ax}}(z)}, \quad \xi_{\text{ax}}(Kz) = \overline{\xi_{\text{ax}}(z)}, \quad \xi_{\text{ax}}(Hz) = \frac{\Phi^2}{\xi_{\text{ax}}(z)}.$$

As real Möbius transformations, $\mathcal{T}$ and H reverse the selected half-plane. Their hyperbolic reflections are the anti-Möbius maps

$$\mathcal{J} = \mathcal{T} \circ K, \quad \mathcal{R}_i = H \circ K.$$

In the $w = \xi_{\text{ax}}(z)$ chart,

$$\mathcal{J} : w \mapsto \frac{1}{\bar{w}}, \quad \mathcal{R}_i : w \mapsto \frac{\Phi^2}{\bar{w}}.$$

Their fixed geodesics are $|w| = 1$ and $|w| = \Phi$, separated by hyperbolic distance $\log \Phi$. Their product is the carriage:

$$\boxed{\mathcal{J}\mathcal{R}_i = L_{\text{ax}}}, \quad w \mapsto \Phi^{-2}w.$$

The reflection $\mathcal{R}_i$ fixes i , because $|\xi_{\text{ax}}(i)| = \Phi$. Moreover

$$\mathcal{J}(i) = 1 + i = L_{\text{ax}}(i).$$

Thus $\mathcal{J}$ does not fix i upstairs, but it fixes the quotient class $[i]$. This is an exact Contact seam: an upstairs Difference closes downstairs without being erased.

RETURN

A typed operator, not a synonym

RETURN is not a universal endomorphism and not a word for resemblance. It is a typed predicate on a declared covering diagram.

Let

$$p : \tilde{X} \longrightarrow X_q$$

be a cover or quotient map. Let $a : X_q \rightarrow X_q$ be an admitted action and $\tilde{a} : \tilde{X} \rightarrow \tilde{X}$ a lift satisfying

$$p \circ \tilde{a} = a \circ p.$$

For $\tilde{x} \in \tilde{X}$, define

$$\boxed{\text{RETURN}_p(\tilde{a}, \tilde{x}) \iff p(\tilde{a}\tilde{x}) = p(\tilde{x}).}$$

RETURN says that the quotient position closes. It does not yet say whether the lift moved. Define the nontrivial lifted clause separately:

$$\boxed{\text{LIFT}_p(\tilde{a}, \tilde{x}; \gamma) \iff \tilde{a}\tilde{x} = \gamma\tilde{x}, \quad \gamma \in \text{Deck}(p), \quad \gamma \neq e.}$$

In the axial realization,

$$\tilde{X} = \mathbb{H}^+, \quad X_q = M_{\text{ax}}, \quad \tilde{a} = L_{\text{ax}}, \quad \gamma = L_{\text{ax}},$$

and the descended action is the identity. Hence

$$p \circ L_{\text{ax}} = p, \quad L_{\text{ax}}z \neq z$$

for every interior point on the axial orbit. The quotient closes; the lift advances.

RETURN = quotient closure + nontrivial lifted Difference + retained relation.

RETURN is not reset.

Memory, address, provenance, receipt

Memory is the compositional relation by which two occurrences can later be compared. It is not a trail effect and not a second copy of the state.

For a declared path or word category $\mathcal{P}$, let

$$\text{MEMORY} : \mathcal{P} \longrightarrow \mathcal{R}$$

be a functor into retained relations, so that

$$\text{MEMORY}(\gamma_2 \circ \gamma_1) = \text{MEMORY}(\gamma_2) \circ \text{MEMORY}(\gamma_1).$$

The axial minimum is the representation

$$\text{MEMORY}_{\text{ax}} : \mathbb{Z} \longrightarrow \text{Deck}(p), \quad n \longmapsto L_{\text{ax}}^n.$$

Its address read is

$$\text{Addr}(L_{\text{ax}}^n z) = n,$$

relative to a declared origin. Its continuous displacement is nT . A word provenance retains the ordered operations that produced the deck element. A receipt appends the occurrence record

$$\text{rcpt}_k = (k, t_k, \text{word}_k, [\text{word}_k], n_k, r_k).$$

The receipt does not rewrite the state, the word, or the global Memory law. These four offices answer four different questions:

Office	Question answered
Address	Where did the lifted traversal arrive?
Provenance	By which ordered operation did it arrive?
Receipt	Which execution occurred, and when was it read?
Memory	Which compositional relation makes occurrences comparable?

a read never overwrites the state it reads.

Recognition at Contact

Let a declared realization be

$$\mathfrak{D} = (X, X_{\text{adm}}, \Delta, \mathcal{A}, \mathcal{I}, q, \tilde{X}, \text{MEMORY}, r).$$

Here X is a carrier; $X_{\text{adm}} \subseteq X$ is the admissible state domain; Δ is oriented Difference; $\mathcal{A}$ contains admitted actions; $\mathcal{I}$ is the invariant or constraint predicate; q is the quotient Reader; $\tilde{X}$ is the lift; MEMORY is the compositional Memory law; and r is a non-overwriting Reader.

For a present Contact

$$c = (\tilde{x}, \tilde{a}, r),$$

define $\text{RECOGNITION}_{\mathfrak{D}}(c) = 1$ when all of the following hold:

1. $\text{RETURN}_q(\tilde{a}, \tilde{x})$ holds;
2. the lifted action differs by a nonidentity deck element;
3. the admitted invariant remains true;
4. Memory retains the ordered relation needed for comparison;
5. the Reader can compare the returned form without mutating its source;
6. address, provenance, and receipt remain separately recoverable.

This is Recognition in its declared operational domain:

Contact recurs through Memory Return.

The manuscript uses the word *Reality* only through the complete predicate

$\text{Reality}_{\mathfrak{D}}(c) := \text{RECOGNITION}_{\mathfrak{D}}(c)$ at present Contact.

Its full language form is:

Reality is present Contact recognizing oriented Difference continuing through Memory Return
inside constraint.

Two shorter sentences are exact projections of this predicate, not substitute definitions:

oriented Difference continuing inside constraint,
carrier/action projection

Motion returns through Memory without erasing Difference .
return/lift projection

The projections are useful only after the complete predicate has fixed their types.

Endurance is non-vacuous

Let $T_n \in \mathcal{A}$ and $x_n \in X_{\text{adm}}$. A traversal endures one step when

$$x_{n+1} = T_n x_n \in X_{\text{adm}}.$$

It endures a word when this remains true at every prefix. This predicate can fail. On the unit timelike shell

$$X_{\text{adm}} = \{v : \eta_G(v, v) = -1\},$$

the full root algebra contains $D = 2I$, but

$$\eta_G(D^n v, D^n v) = -4^n$$

for every $n \geq 1$. The action leaves the selected shell immediately and does not return. By contrast, $L_{\text{ax}} = e^{(\log \Phi)S}$ preserves η_G and its entire orbit remains admissible.

This distinction is elementary but exact. The root algebra contains both lawful and inadmissible actions relative to a selected realization; membership in the complete algebra alone does not certify survival inside a constraint.

ONE CARRIAGE, TWO READS

Three representations of one element

The economy of the construction is not that several offices are renamed as one thing. It is that one element acts honestly in three typed representations.

Representation	Exact action	What is preserved
Linear on (V, η_G)	Lorentz boost of rapidity $\log \Phi$	signed form and null rays
Projective on $\mathbb{H}^+$	hyperbolic transvection of length $2 \log \Phi$	Poincaré metric and golden axis
Deck action on M_{ax}	one lifted address step	quotient class and harmonic period

The linear statement is

$$L_{\text{ax}}^T G L_{\text{ax}} = G.$$

The projective statement follows because $\det L_{\text{ax}} = 1$, so its Möbius action is an orientation-preserving hyperbolic isometry. The deck statement is

$$p \circ L_{\text{ax}} = p.$$

A deck transformation is not automatically holonomy. Holonomy requires a declared connection and parallel transport. No such connection is smuggled into the word *return*; the exact terms here are boost, transvection, and deck action.

One coframe

Choose a g_i -orthonormal eigenframe (e_0, e_1) of H with

$$H e_0 = -e_0, \quad H e_1 = e_1.$$

Then

$$g_i(e_a, e_b) = \delta_{ab}, \quad \eta_G(e_0, e_0) = -1, \quad \eta_G(e_1, e_1) = 1, \quad \eta_G(e_0, e_1) = 0.$$

For a supplied positive scale ℓ , define the V -valued coframe

$$\Theta = \ell(e_1 d\delta + e_0 \cosh \delta d\Lambda).$$

The coframe has two exact pullbacks. Through the positive metric,

$$\Theta^* g_i = \ell^2(d\delta^2 + \cosh^2 \delta d\Lambda^2).$$

Through the signed metric,

$$\Theta^* \eta_G = \ell^2(d\delta^2 - \cosh^2 \delta d\Lambda^2).$$

No metric has been swapped out. The same coframe is read once by the positive form and once by the signed form. The first is the hyperbolic-plane metric of curvature $-\ell^{-2}$. The second is the global static form of $\widehat{\text{AdS}}_2(\ell)$, also of curvature $-\ell^{-2}$, with unwrapped time coordinate $\Lambda \in \mathbb{R}$.

The causal wall

The positive projective metric admits the quotient

$$\Lambda \sim \Lambda + T.$$

Applying the same identification to the Lorentzian pullback would make every constant- δ orbit a closed timelike curve, because

$$ds_L^2|_{\delta=\text{const}} = -\ell^2 \cosh^2 \delta d\Lambda^2 < 0.$$

Therefore the two reads separate at topology:

positive projective read	Lorentzian physical read
$\Lambda \in \mathbb{R}/T\mathbb{Z}$	$\Lambda \in \mathbb{R}$
quotient RETURN	universal-cover Time
$[\tilde{\alpha}_{\text{ret}}] \neq 0$	no timelike deck identification

The universal cover removes the periodic-time closed curves. It does not make anti-de Sitter space globally hyperbolic: the conformal boundary is timelike. Any global evolution problem requires boundary conditions or a globally hyperbolic subregion. No stronger causal claim is made.

The marked isometry

$$\Lambda \longmapsto \Lambda + T, \quad T = 2 \log \Phi,$$

remains part of the root-to-spacetime adapter. The unmarked local AdS_2 metric by itself does not remember Φ .

Light and Time

The signed root carrier has two null lines,

$$\eta_G(v_\Phi, v_\Phi) = 0, \quad \eta_G(v_\Psi, v_\Psi) = 0.$$

The boost acts on them by reciprocal positive scale:

$$L_{\text{ax}} v_\Phi = \Phi v_\Phi, \quad L_{\text{ax}} v_\Psi = \Phi^{-1} v_\Psi.$$

This earns a minimum Light office: null directions of an indefinite form are carried without leaving the null boundary. It does not yet supply a Maxwell field, polarization, energy, or a measured propagation law. Those require a physical realization.

The Lorentzian coframe earns a minimum Time office: Λ is the unbounded timelike coordinate of the signed read, and ∂_Λ is its timelike Killing field. It is not the quotient clock. The positive quotient measures return class; the Lorentzian cover orders continuing motion.

This is why the architecture does not assign one coordinate two incompatible topologies. Memory closes the positive projective position. Time retains the unquotiented order.

Noether after an action

Symmetry alone is not yet a Noether theorem. Supply the geodesic action on the Lorentzian read,

$$\mathcal{S}_{\text{geo}} = \frac{1}{2} \int \ell^2 (\dot{\delta}^2 - \cosh^2 \delta \dot{\Lambda}^2) ds.$$

The Lagrangian is independent of Λ . Only now does Noether's theorem apply, giving the conjugate momentum

$$p_{\Lambda} = \frac{\partial \mathcal{L}}{\partial \dot{\Lambda}} = -\ell^2 \cosh^2 \delta \dot{\Lambda}, \quad \frac{dp_{\Lambda}}{ds} = 0.$$

The root supplies the axial Killing direction. The action supplies the dynamical hypothesis. The conserved quantity follows. This ordering is the general rule of the paper: a classical theorem enters at its native office and under its native assumptions.

PHYSICAL SEALS

A seal is a native test

A physical seal neither generates the root nor merely resembles it. It supplies target-side data, applies an explicit adapter, and tests the resulting object in the target theory's own equation or inequality.

$$\boxed{\text{root data} \xrightarrow{\text{adapter}} \text{realization candidate} \xrightarrow{\text{native constraint}} \text{admitted or refused.}}$$

Four claim statuses are used:

Status	Meaning
CERTIFIED	exact root identity or theorem, checked in its native domain
DERIVED	exact consequence using a displayed adapter
SUPPLIED	standard target-side datum not generated by the root
OPEN	candidate whose required adapter or empirical test is incomplete

No claim crosses a status boundary silently.

Gravity seal: three exact layers

Layer 1: root Lorentz carrier

The root itself supplies

$$(V, \eta_G), \quad S \in \mathfrak{so}(\eta_G), \quad L_{\text{ax}} = e^{(\log \Phi)S},$$

with golden null rays and a marked boost. This layer is CERTIFIED from (C_1, Q) .

Layer 2: axial solder

The coframe Θ supplies the exact pullback

$$\Theta^* \eta_G = \ell^2 (d\delta^2 - \cosh^2 \delta d\Lambda^2).$$

This is a DERIVED adapter to $\widetilde{\text{AdS}}_2(\ell)$, marked by the isometry $\Lambda \mapsto \Lambda + 2 \log \Phi$.

Layer 3: four-dimensional completion

Supply an equal-radius sphere and Maxwell flux. In one standard normalization,

$$\mathcal{M}_{\text{BR}} = \widetilde{\text{AdS}}_2(\ell) \times S^2(\ell),$$

$$g_4 = \ell^2 (d\delta^2 - \cosh^2 \delta d\Lambda^2) + \ell^2 d\Omega_2^2,$$

and, for an orthonormal coframe θ^0, θ^1 on the Lorentz factor,

$$F = \ell^{-1} \theta^0 \wedge \theta^1.$$

With the declared unit convention, the pair satisfies the source-free Einstein–Maxwell system

$$\mathcal{G}_{\mu\nu} = 2 \left(F_{\mu\rho} F_{\nu}{}^{\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right), \quad dF = d*F = 0.$$

This is the Bertotti–Robinson realization. The solution is standard; the paper’s contribution is the exact typed route by which the root’s signed carrier and marked axial motion enter its AdS_2 factor.

The root does not generate $S^2(\ell)$, the scale ℓ , Maxwell flux, Newton’s constant, boundary conditions, or the Einstein equations. The completion is SUPPLIED and then passes its native equation exactly. In two dimensions the Einstein tensor vanishes identically; the sphere and flux are real completion data, not decoration.

Quantum seal: the welded Bell frame

Root-side observable pair

Complexify the real carrier:

$$\mathcal{H}_q = V \otimes_{\mathbb{R}} \mathbb{C},$$

with the Hermitian extension of g_i . Because H and S are real symmetric involutions, they become Hermitian dichotomic observables on $\mathcal{H}_q$:

$$H^\dagger = H, \quad S^\dagger = S, \quad H^2 = S^2 = I, \quad \{H, S\} = 0.$$

In Pauli form,

$$S = \frac{\sigma_z + 2\sigma_x}{\sqrt{5}}, \quad H = \frac{-2\sigma_z + \sigma_x}{\sqrt{5}}.$$

The Fibonacci transport has therefore selected one exact orthogonal pair in the planar Clifford family. Tsirelson saturation is generic to anticommuting qubit observables; it is not a new golden constant. What is root-specific is that this particular pair is generated internally by

$$G = [C_1, Q], \quad H = G/\sqrt{5}, \quad S = HC_1.$$

Supplied singlet realization

Supply the bipartite carrier

$$\mathcal{H}_{AB} = \mathcal{H}_q \otimes \mathcal{H}_q$$

and the singlet state

$$\rho_- = |\psi^-\rangle\langle\psi^-|.$$

Define settings

$$A_0 = S, \quad A_1 = H, \quad B_0 = \frac{S+H}{\sqrt{2}}, \quad B_1 = \frac{S-H}{\sqrt{2}}.$$

Anticommutation gives $B_0^2 = B_1^2 = I$. The supplied singlet Reader is

$$E(A, B) = \text{Tr}(\rho_- A \otimes B) = -\frac{1}{2} \text{tr}(AB)$$

for these planar traceless observables. Hence

$$(E_{00}, E_{01}, E_{10}, E_{11}) = \frac{1}{\sqrt{2}}(-1, -1, -1, +1),$$

and

$$\boxed{\mathcal{S}_{\text{CHSH}} = E_{00} + E_{01} + E_{10} - E_{11} = -2\sqrt{2}.}$$

Under locality/factorization and measurement independence, any local hidden variable model obeys $|\mathcal{S}_{\text{CHSH}}| \leq 2$. The declared realization refuses that flattening exactly.

The golden operator read

The root also reconstructs the fixed oriented frame directly:

$$Z = \frac{S - 2H}{\sqrt{5}} = \sigma_z, \quad X = \frac{2S + H}{\sqrt{5}} = \sigma_x.$$

Use

$$A_0 = Z, \quad A_1 = X, \quad B_0 = S, \quad B_1 = -H.$$

Then

$$(E_{00}, E_{01}, E_{10}, E_{11}) = \frac{1}{\sqrt{5}}(-1, -2, -2, +1),$$

so

$$\boxed{|\mathcal{S}_{\sqrt{5}}| = \frac{6}{\sqrt{5}}, \quad 2 < \frac{6}{\sqrt{5}} < 2\sqrt{2}.}$$

This value depends on the displayed setting and outcome convention. It is an exact root-indexed Reader value, not a universal physical constant.

Presence, Contact, Reader, evidence

The full experimental Presence is not the scalar correlation. It is the apparatus tuple

$$P = (\rho_-; A_0, A_1; B_0, B_1; \{\mathcal{I}_{st|xy}\}),$$

where $\mathcal{I}_{st|xy}$ are declared joint instruments. Contact is the state-instrument interaction. The Reader is

$$r_{xy}(P) = \sum_{s,t=\pm 1} st \text{Tr} \mathcal{I}_{st|xy}(\rho_-).$$

The local record (x, y, s, t) is an evidence receipt. It is neither the state nor the global compositional law by which repeated trials are compared. This typing is part of the realization, not editorial decoration.

The root supplies a canonical anticommuting observable pair and an exact golden setting frame. It does not supply the singlet, tensor product, Born rule, laboratory independence assumptions, or instruments. Those are SUPPLIED; the Bell inequality is the native CONSTRAINT; the correlations are READ.

What the seals establish

Seal	Root contribution	Supplied data	Native test
Hodge/de Rham	axial quotient and $d\Lambda$	quotient orientation	harmonic, nonexact period class
Noether Gravity	Killing field ∂_Λ G, H, S, L_{ax}, Θ	geodesic action $S^2(\ell)$, flux, field equations	conserved p_Λ Einstein–Maxwell admits
Quantum	anticommuting H/S frame	singlet, instruments, Born Reader	Bell–CHSH refuses flattening

The seals do not say that Hodge theory, Noether’s theorem, General Relativity, and quantum mechanics are one downstream object. They show that each native lineage admits a structure-preserving realization of the same return diagram:

carrier $\longrightarrow$ oriented action $\longrightarrow$ invariant $\longrightarrow$ quotient | lift $\longrightarrow$ Reader and retained evidence.

Universality here means dynamical identity, not object identity. Every realization admitted by this manuscript is tested against that declared contract. A stronger classification theorem for every possible physical or formal system is not used as a premise.

CLOSURE

The diamond in one line

The exact architecture is now short enough to carry without compression loss:

$$\begin{array}{l}
 (C_1, Q) \longrightarrow G = [C_1, Q] \\
 \longrightarrow H = G / \sqrt{5}, \quad S = HC_1 \\
 \longrightarrow L_{\text{ax}} = e^{(\log \Phi)S} \\
 \xrightarrow{\xi_{\text{ax}}} w \mapsto \Phi^{-2}w \\
 \xrightarrow{\Lambda = \log(\Phi/|w|)} \Lambda \mapsto \Lambda + 2 \log \Phi \\
 \xrightarrow{\text{quotient}|\text{cover}} \text{Return} \mid \text{continuing address} \\
 \xrightarrow{\text{Contact}} \text{Recognition.}
 \end{array}$$

The coordinate i is still while the address grows:

$$\Lambda(i) = 0, \quad [L_{\text{ax}}^n i] = [i], \quad \Lambda(L_{\text{ax}}^n i) = nT.$$

The first equality fixes the reference. The second is quotient Return. The third is lifted growth. None cancels another.

What is original, what is inherited

The individual ingredients of the construction belong to classical mathematics and physics: alternating forms, complex structures, Fibonacci transport, Lorentz boosts, hyperbolic geometry, covering spaces, harmonic forms, Noether symmetry, Bell inequalities, and the Bertotti–Robinson solution. The claimed contribution is their exact composite construction from the displayed root and the type-separated route by which each realization is admitted.

The paper’s formal knife is the refusal to let closure erase traversal. That single obstruction organizes the offices:

- the two-form retains oriented relation before object identity;
- the root wheel fixes the minimum turn;
- the commutator and traceless transport weld into $S = HC_1$;
- the positive carriage isolates forward motion from orientation reversal;
- the golden chart rectifies curvature without denying it;
- the quotient closes position while the cover retains order;
- the harmonic class retains a global period;
- the Reader observes without overwriting the state;
- a physical seal is accepted only in its native constraint.

Final form

A form can return without ending.

Its quotient closes.

Its lift continues.

Its Difference remains readable.

THIS IS LINEAR INFINITY.

REFERENCE CARDS

Exact normal-form card

Office	Exact normal form
Carrier	$V = \mathbb{R}^2$
Oriented form	$\omega(u, v) = u^\top \Omega v, \quad \Omega = -C_1$
Quarter-turn	$C_1^2 = -I, \quad C_1^4 = I$
Transport	$Q^2 = Q + I, \quad \det Q = -1$
Difference	$G = [C_1, Q], \quad G^2 = 5I$
Signed unit	$H = G/\sqrt{5}, \quad H^2 = I$
Boost generator	$S = HC_1 = (2Q - I)/\sqrt{5}, \quad S^2 = I$
Weld	$HS = C_1, \quad SH = -C_1, \quad \{H, S\} = 0$
Carriage	$L_{\text{ax}} = (Q + 2I)/\sqrt{5} = e^{(\log \Phi)S}$
Two-step lift	$L_{\text{ax}}^2 = Q^2$
Signed invariance	$L_{\text{ax}}^\top G L_{\text{ax}} = G$
Golden boundary	$\Phi, \Psi; \quad \Phi + \Psi = 1, \quad \Phi\Psi = -1$
Rectifier	$\xi_{\text{ax}}(z) = (\Phi - z)/(z - \Psi)$
Contact	$\xi_{\text{ax}}(i) = -i\Phi, \quad \Lambda(i) = 0$
Linear coordinate	$\Lambda = \log(\Phi/ \xi_{\text{ax}})$
Translation	$\Lambda(L_{\text{ax}}z) = \Lambda(z) + T, \quad T = 2 \log \Phi$
Rail	$\delta = \operatorname{arsinh}(X/\hat{Y}) = 0$
Positive read	$ds_H^2 = d\delta^2 + \cosh^2 \delta d\Lambda^2$
Lorentz read	$ds_L^2 = d\delta^2 - \cosh^2 \delta d\Lambda^2$
Memory class	$p^* \tilde{\alpha}_{\text{ret}} = d\Lambda, \quad \oint \tilde{\alpha}_{\text{ret}} = T$
Return	$p(L_{\text{ax}}z) = p(z), \quad L_{\text{ax}}z \neq z$

Proof dependency and status card

Claim	Status	Dependency
Root algebra and wheel	CERTIFIED	(C_1, Q) only
Weld $S = HC_1$	CERTIFIED	matrix multiplication
Positive carriage	CERTIFIED	root identities and golden algebra
Golden conjugacy	CERTIFIED	projective calculation
Linear Infinity theorem	DERIVED	conjugacy, log coordinate, quotient
Harmonic Memory class	DERIVED	explicit Fermi metric and quotient
Lorentz coframe	DERIVED	H -eigenframe and supplied scale
Noether momentum	DERIVED	supplied geodesic action
Bertotti–Robinson seal	SUPPLIED + DERIVED	axial solder plus S^2 , flux, equations
Bell maximal seal	SUPPLIED + DERIVED	H/S plus singlet/Born Reader
Golden Bell read	SUPPLIED + DERIVED	displayed setting convention
Empirical constants or particles	OPEN unless separately certified	no premise of the main theorem

Standing type walls

1. G as endomorphism and $u^T G v$ as its basis-dependent bilinear presentation remain typed.
2. Raw linear Euclidean action, projective hyperbolic action, Lorentz action, and deck action are distinct representations.
3. The positive quotient is a cylinder; Lorentzian Time remains on the universal cover.
4. $d\Lambda$ is exact on the cover; the descended harmonic form is nonexact on the quotient.
5. A deck action is not called holonomy without a connection.
6. Noether enters only after an action has been supplied.
7. The root supplies neither laboratory state nor four-dimensional completion data.
8. A Reader never overwrites the state it reads.

Companion finite-address theorem: 137 and 548

This appendix is compatible with Linear Infinity but is not a premise of it. It records the finite wheel and mirror address supplied by the same root.

The quarter-turn order is forced:

$$m = 4.$$

Two antecedent Reader choices fix a complete traversal at $s = m + 1 = 5$ observed states and a centered inverse-readable window at $r = 2m + 1 = 9$ marks. With F_n the Fibonacci sequence and $L_n = \text{tr}(Q^n)$ the Lucas sequence,

$$L_5 = 11, \quad F_9 = 34.$$

The two routes converge:

$$137 = L_5^2 + m^2 = 11^2 + 4^2 = 1 + mF_9 = 1 + 4 \cdot 34.$$

The value is not certified merely by that coincidence. It passes the finite mirror closure:

$$4^{17} \equiv -1 \pmod{137}, \quad 4^{34} \equiv 1 \pmod{137},$$

with no proper divisor of 34 returning 1. Hence

$$\text{ord}_{137}(4) = 34.$$

Since $\mathbb{F}_{137}^\times$ is cyclic of order $136 = 4 \cdot 34$,

$$1 \longrightarrow C_{34} \longrightarrow \mathbb{F}_{137}^\times \longrightarrow C_4 \longrightarrow 1.$$

Moreover, 137 is the least prime for which 4 has multiplicative order 34. The only smaller prime congruent to 1 (mod 34) is 103, and

$$4^{34} \equiv 56 \pmod{103}.$$

Thus the finite construction closes the order-34 carriage with an order-four quotient. Combining the coprime wheel and mirror addresses gives

$$548 = 4 \cdot 137, \quad \mathbb{Z}/548\mathbb{Z} \cong \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/137\mathbb{Z}.$$

The competing value 1277 from a different “fresh invariant” Reader does not satisfy this declared mirror criterion:

$$\text{ord}_{1277}(4) = 638,$$

and its quotient order is 2, not 4. The reader selection is therefore not “whichever invariant produces 137.” It is the explicitly stated four-wheel/order-34/four-quotient closure.

The 2022 CODATA low-energy inverse electromagnetic fine-structure constant is $\alpha^{-1}(0) = 137.035\,999\,177(21)$ [14], whose nearest integer is 137. This is recorded as a physical Recognition of the finite address, not as a derivation of the measured constant. Running with energy, nuclear

identification, and Higgs-sector identification require separate physical adapters and are not claims of this theorem.

Exact verification ledger

All root, carriage, signed-form, projective, finite-address, and additional Linear Infinity identities were checked in exact rational $\mathbb{R}(\sqrt{5})$ arithmetic. Floating point is not used as evidence.

The established spine verifier checks, among other statements,

- $C_1^2 = -I$, $Q^2 = Q + I$, and $G^2 = 5I$;
- one-step form reversal and two-step preservation;
- $M_0^2 = I$, $\det L_{\text{ax}} = 1$, and $L_{\text{ax}}^2 = Q^2$;
- $S^2 = I$, $S^T G + G S = 0$, and $e^{(\log \Phi)S} = L_{\text{ax}}$;
- golden null directions and reciprocal eigen-scaling;
- the ξ_{ax} conjugacies for M_0, H, L_{ax} ;
- the finite 137/548 order and Chinese-remainder claims;
- the unit Bertotti–Robinson tensor normalization.

The Linear Infinity companion verifier checks

- $GC_1 = 2Q - I$, $S = HC_1$, $SH = -C_1$, $HS = C_1$;
- the H/S eigenframes and their Hadamard overlap;
- the exact squared Bell values 8 and $4(1 + 2/\sqrt{5})$ for the declared Readers;
- a permanent failure of the unit-shell Endurance predicate under $D = 2I$.

The verification scripts accompany this edition:

```
verify_exact_spine.py
verify_linear_infinity.py
```

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THE QUOTIENT RETURNS. THE LIFT CONTINUES.