

Scope Recognition: Metric Minimum

Reference, Return & Residual in the Relativistic Field Metric

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Abstract

A measurement becomes repeatable when its reference, reader, path, oriented rotation, resolution, and remaining mismatch are carried with its reported value. This paper gives an exact architecture for that requirement. Its root is the integer Fibonacci carriage Q , whose commutator with the oriented two-form Ω produces a signature- $(1, 1)$ form B . In null coordinates the form is $-2\xi\eta$, the same Lorentzian normal form as the (u, v) sector of relativistic spacetime.

A return reader, lift, and occurrence record distinguish a closed excursion from a constant path when their displayed reads agree. Cup symmetry supplies the constrained-motion and Noether-current forms; the axis of $Q^2 \in \text{PSL}_2(\mathbb{Z})$ supplies their standard hyperbolic realization. A finite K_8 carrier audits its twenty-eight addressed seams through fifty-six triangle closures. An Einstein–Maxwell pp-wave supplies a native field realization. A CIE-standard eye reader supplies the embodied detector-side measurement map. The Higgs field supplies the gauge-invariant scalar identity read and its metric stress-energy scope.

The result is a scope-recognition discipline: every report retains its carrier, reader, invariant, resolution, residual, and record through physical advance.

1. Addressed measurement

At a physical contact event q_n , define

$$\mathcal{R}_\rho(q_n) = (\mathcal{D}, \mathcal{C}, \rho; q_n, \alpha_n, (p_n, w_n); B; g, A, F, \mathcal{H}; \epsilon_n, \Pi_n). \quad (1)$$

Entry	Office
$\mathcal{D}$	Chart, units, boundary data, and domain of validity.
$\mathcal{C}$	Reference system and comparison contract.
ρ	Observer, instrument, or calibrated protocol.
q_n	Physical event of contact.
$\alpha_n, (p_n, w_n)$	Finite address, phase, and winding count.
B	Fixed signed-difference form of the root plane.
$g, A, F, \mathcal{H}$	Metric, gauge potential, field strength, and Higgs field.
ϵ_n	Residual in a declared norm.
Π_n	Inputs, derivation, calibration, computational method, and checks.

A record closes at declared resolution δ when every entry is declared and

$$\epsilon_n \leq \delta. \quad (2)$$

This closes the stated comparison. The physical carrier continues through its admitted field equations.

For a return reader $r : X \rightarrow Y$,

$$r(x(1)) = r(x(0)) \quad (3)$$

holds in Y . A constant path and a closed excursion can therefore share the same displayed read. Lift and record preserve their addressed difference.

2. Root metric and the common null normal form

On $V = \mathbb{R}^2$, let

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = [Q, \Omega] = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}. \quad (4)$$

The root identities are

$$B^2 = 5I, \quad Q^\top BQ = -B, \quad \Omega^\top B\Omega = -B. \quad (5)$$

Set

$$H_B = \frac{B}{\sqrt{5}}, \quad S = \frac{2Q - I}{\sqrt{5}}, \quad L_s = e^{sS}. \quad (6)$$

Then

$$H_B^2 = S^2 = I, \quad S^\top B + BS = 0, \quad H_B S = -S H_B, \quad \Omega = S H_B, \quad (7)$$

and

$$L_s^\top B L_s = B, \quad H_B L_s H_B = L_{-s}. \quad (8)$$

Let $\varphi = (1 + \sqrt{5})/2$. For $z = (a, b)^\top$, define

$$\xi(z) = a - \varphi b, \quad \eta(z) = a + \varphi^{-1} b. \quad (9)$$

The root metric factors as

$$z^\top B z = -2\xi(z)\eta(z). \quad (10)$$

Its null lines are the golden eigendirections of Q , with slopes φ^{-1} and $-\varphi$. The reflected-axis contact is

$$\arctan(\varphi^3) - \arctan(\varphi^{-1}) = \frac{\pi}{4}. \quad (11)$$

In relativistic coordinates

$$u = X^0 - X^2, \quad v = \frac{1}{2}(X^0 + X^2), \quad (12)$$

the physical interval is

$$-(dX^0)^2 + (dX^2)^2 = -2 du dv. \quad (13)$$

Thus the root and physical sectors carry explicitly isomorphic Lorentzian two-planes:

$$(\xi, \eta) \longleftrightarrow (u, v). \quad (14)$$

3. Motion, return, lift, and record

The exact Memory Return datum is

$$\mathbf{MR} = (r, x, p, \gamma, \tilde{\gamma}, \varepsilon, \eta_{\text{act}}, \rho_{\text{rep}}, D_{\rho_{\text{rep}}}(\varepsilon), R). \quad (15)$$

Here $p : \tilde{B} \rightarrow B$ is a declared lift system, $\tilde{\gamma}(1) = \varepsilon \tilde{\gamma}(0)$ records the lift endpoint defect, $D_{\rho_{\text{rep}}}(\varepsilon)$ is its represented difference, and R retains the occurrence-bearing record. The symbol ρ_{rep} names a group representation; the observer protocol in (1) remains ρ .

For $v_{n+1} = Qv_n$,

$$(-1)^n v_n^T B v_n = v_0^T B v_0, \quad (Q^2)^* B = B. \quad (16)$$

Where $\xi\eta \neq 0$, the axial coordinate

$$\lambda(z) = \log \left| \frac{\eta(z)}{\xi(z)} \right| \quad (17)$$

obeys

$$\lambda(L_s z) = \lambda(z) + 2s. \quad (18)$$

For occurrences $o_n = (\mathbf{u}, n)$,

$$p_{n+4} = p_n, \quad o_{n+4} \neq o_n, \quad R_{n+4} \succ R_n. \quad (19)$$

Motion therefore carries an excursion; the reader supplies the return; the lift and record hold the advance.

4. Cup symmetry and its modular realization

A cup datum is

$$\mathbf{C} = (M, g_R, C, I, \gamma, \kappa_{\text{max}}, r, \eta_{\text{return}}), \quad (20)$$

where $\gamma : I \rightarrow C \subseteq M$ is regular, $0 \leq \kappa_\gamma \leq \kappa_{\text{max}} < \infty$, and η_{return} witnesses reader-return. A mirror cup satisfies

$$K_C^2 = \text{id}_C, \quad K_C \zeta_C K_C = \zeta_C^{-1}. \quad (21)$$

For infinite-order transport,

$$\langle \zeta_C, K_C \rangle \cong D_\infty. \quad (22)$$

The modular realization is

$$Q^2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \in \mathrm{PSL}_2(\mathbb{Z}), \quad \mathrm{tr}(Q^2) = 3. \quad (23)$$

Its axis in the upper half-plane is

$$\left| z - \frac{1}{2} \right| = \frac{\sqrt{5}}{2}, \quad (24)$$

with ideal endpoints φ and $-\varphi^{-1}$. Hyperbolic elements of $\mathrm{PSL}_2(\mathbb{Z})$ have integral trace of absolute value at least three; (23) therefore has minimum hyperbolic translation length and projects to the shortest closed modular geodesic.

The modular mirror is

$$\mathcal{K}_{\mathrm{mod}} = \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}, \quad \mathcal{K}_{\mathrm{mod}}^2 = I, \quad \mathcal{K}_{\mathrm{mod}} Q^2 \mathcal{K}_{\mathrm{mod}} = (Q^2)^{-1}. \quad (25)$$

H_B and $\mathcal{K}_{\mathrm{mod}}$ are separate realizations of the inversion office. Their common golden discriminant is $\sqrt{5}$; the sharp Hurwitz approximation coefficient is $1/\sqrt{5}$.

When a one-parameter variational symmetry preserves a cup and its admissible paths, Noether's theorem supplies a conserved current. In the radial mechanical cup,

$$L_z = mR^2\dot{\theta}, \quad \frac{dL_z}{dt} = 0. \quad (26)$$

5. Finite compatibility carrier

Let eight occurrence addresses form K_8 . Their twenty-eight seams have coefficients $d_{ij} = -d_{ji}$. Each oriented triangle carries

$$h_{ijk} = d_{ij} + d_{jk} + d_{ki}. \quad (27)$$

The complete finite criterion is

$$\forall i < j < k, \ h_{ijk} = 0 \iff \exists (s_0, \dots, s_7), \ d_{ij} = s_j - s_i. \quad (28)$$

The seven matchings

$$M_r = \{\{\infty, r\}\} \cup \{\{r+j, r-j\} : j = 1, 2, 3\}, \quad r \in \mathbb{Z}_7, \quad (29)$$

partition $E(K_8)$. The fifty-six triangle faces audit every addressed comparison. Vanishing face residuals give a global potential read.

6. Einstein–Maxwell null realization

Take

$$(X^0, X^1, X^2, X^3) = (ct, \rho\vartheta, x, y), \quad X^1 \sim X^1 + 2\pi\rho. \quad (30)$$

The field equations are

$$F = dA, \quad dF = 0, \quad d \star_g F = 0, \quad (31)$$

$$G_{\mu\nu}[g] + \Lambda_{\text{cos}} g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}. \quad (32)$$

On a fixed flat background, set

$$A = \frac{a_{\text{hol}}}{\rho} dX^1 + f(u) dy, \quad F = q(u) du \wedge dy, \quad q = f'. \quad (33)$$

This is a source-free null Maxwell wave. Its compact loop read is

$$\oint_{\ell} A = 2\pi a_{\text{hol}}. \quad (34)$$

The self-gravitating pp-wave realization is an Einstein–Maxwell realization of (32) with

$$\Lambda_{\text{cos}} = 0, \quad T_{\mu\nu} = T_{\mu\nu}^{\text{EM}}. \quad (35a)$$

It is

$$ds^2 = -2dudv + (dX^1)^2 + dy^2 - \kappa_{\text{EM}} q(u)^2 y^2 du^2, \quad F = q(u) du \wedge dy, \quad (35)$$

with $\kappa_{\text{EM}} = 8\pi G_N/c^4$. Under the declared normalization,

$$T_{uu}^{\text{EM}} = q(u)^2, \quad R_{uu} = \kappa_{\text{EM}} q(u)^2, \quad R = 0, \quad \mathcal{E}_{\mu\nu}[g, A; \Lambda_{\text{cos}} = 0] = 0. \quad (36)$$

7. Higgs identity and metric scope

The Higgs field is denoted by $\mathcal{H}$, preserving H_B for the root-plane mirror. In the electroweak field sector with declared $SU(2)_L \times U(1)_Y$ connection,

$$\mathcal{H} \in \mathbb{C}^2, \quad \mathcal{I}_{\mathcal{H}} := \mathcal{H}^\dagger \mathcal{H}. \quad (37)$$

$\mathcal{I}_{\mathcal{H}}$ is the local gauge-invariant scalar read of the Higgs field. The covariant derivative D_μ carries the declared electroweak connection. With metric signature $(-+++)$, its action density is

$$\mathcal{L}_{\mathcal{H}} = -g^{\mu\nu} (D_\mu \mathcal{H})^\dagger D_\nu \mathcal{H} - V(\mathcal{H}), \quad V(\mathcal{H}) = -\mu^2 \mathcal{H}^\dagger \mathcal{H} + \lambda (\mathcal{H}^\dagger \mathcal{H})^2. \quad (38)$$

For $\mu^2, \lambda > 0$, the vacuum modulus is

$$\mathcal{H}^\dagger \mathcal{H} = \frac{\mu^2}{2\lambda}. \quad (39)$$

The metric carries the Higgs kinetic relation; variation with respect to $g^{\mu\nu}$ supplies $T_{\mu\nu}^{\mathcal{H}}$, which enters the metric equation through

$$G_{\mu\nu}[g] + \Lambda_{\text{cos}} g_{\mu\nu} = \frac{8\pi G_N}{c^4} (T_{\mu\nu}^{\text{EM}} + T_{\mu\nu}^{\mathcal{H}} + T_{\mu\nu}^{\text{other}}). \quad (40)$$

For a constant electroweak vacuum, $D_\mu \mathcal{H} = 0$,

$$T_{\mu\nu}^{\mathcal{H}} = -g_{\mu\nu} V_{\text{vac}}, \quad V_{\text{vac}} = -\frac{\mu^4}{4\lambda}. \quad (40a)$$

Thus the pp-wave metric of (35) enters the full metric equation precisely under the declared vacuum condition

$$\Lambda_{\text{cos}} + \frac{8\pi G_N}{c^4} V_{\text{vac}} = 0. \quad (40b)$$

The Einstein–Maxwell witness in §6 uses the equivalent zero-vacuum-energy convention

$$\widetilde{V}(\mathcal{H}) = V(\mathcal{H}) - V_{\text{vac}}, \quad \widetilde{V}(\mathcal{H}_{\text{vac}}) = 0, \quad \Lambda_{\text{cos}} = 0. \quad (40c)$$

The Higgs entry in the measurement record is therefore

$$\text{Scope}_{\mathcal{H}}(q) = (\mathcal{H}|_q, D\mathcal{H}|_q, \mathcal{H}^\dagger \mathcal{H}|_q, T^{\mathcal{H}}|_q, g|_q). \quad (41)$$

It is the field-theoretic minimum identity scope: a gauge-invariant local scalar, its covariant transport, its energy-momentum relation, and the metric that carries their physical comparison.

8. Eye reader at null contact

Let γ_{null} be a declared null-flow segment and Γ_{eye} the observer’s timelike worldline. The detector-side contact is

$$q_n = \gamma_{\text{null}}(\lambda_n) \cap \Gamma_{\text{eye}}(\tau_n). \quad (42)$$

The eye reader is the specialized observer/protocol entry of (1):

$$\rho_{\text{eye}} = (\Gamma_{\text{eye}}, r_{\text{eye, CIE}}, \text{optical and retinal response, calibration}, \delta_{\text{eye}}, \Pi_{\text{eye}}). \quad (43)$$

The CIE-standard output map is

$$r_{\text{eye, CIE}} : (\text{incident spectrum at } q_n, \text{path, geometry, observer functions}) \mapsto (X, Y, Z)_n. \quad (44)$$

For spectral distribution L_λ ,

$$(X, Y, Z) = k \int L_\lambda(\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)) d\lambda, \quad (45)$$

with the selected observer functions, geometry, normalization, and calibration recorded in Π_{eye} .

Equal visual reads satisfy

$$r_{\text{eye, CIE}}(q_{n+1}) = r_{\text{eye, CIE}}(q_n) \quad (46)$$

in their declared output space. Their distinct events, paths, phase/winding addresses, and records remain present in $\mathcal{R}_{\rho_{\text{eye}}}$.

9. Covariant axial face charge

For the Einstein–Maxwell action

$$\mathcal{S}[g, A] = \int \left[\frac{c^3}{16\pi G_N} (R - 2\Lambda_{\text{cos}}) - \frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} \right] \sqrt{-g} d^4x, \quad (47)$$

a declared pair (ξ, χ) gives

$$\delta g = \mathcal{L}_\xi g, \quad \delta A = \mathcal{L}_\xi A + d\chi. \quad (48)$$

The following is the Einstein–Maxwell current subaction. The full metric–Higgs current is obtained by adjoining $\mathcal{L}_{\mathcal{H}}$ and the electroweak gauge sector from (38).

The covariant Noether construction is

$$\mathcal{J}_{\xi, \chi} = \vartheta(\phi, \delta_{\xi, \chi} \phi) - \iota_\xi L, \quad d\mathcal{J}_{\xi, \chi} = 0 \quad \text{on shell.} \quad (49)$$

Fix the orientation

$$\epsilon = du \wedge dv \wedge dX^1 \wedge dy, \quad (50)$$

and use the Einstein–Maxwell charge-two-form convention

$$\mathcal{Q}_{\xi, \chi}^N = -\frac{c^3}{16\pi G_N} \star_g d\xi^\flat - \frac{1}{\mu_0} (\iota_\xi A + \chi) \star_g F. \quad (51)$$

For the pp-wave of (35), the compact axial symmetry is

$$(\xi_{\text{ax}}, \chi_{\text{ax}}) = (\partial_{X^1}, 0), \quad (52)$$

since the declared g and A are independent of X^1 . The finite triangle σ_{ijk} remains its own combinatorial carrier. Its physical placement is an oriented axial two-chain tagged by that face,

$$\Sigma_{ijk} = I_{ijk} \times J_{ijk}, \quad \iota_{ijk}(u, X^1) = (u, v_{ijk}, X^1, y_{ijk}), \quad du \wedge dX^1 > 0, \quad (53)$$

where $I_{ijk} = [u_{ijk}^-, u_{ijk}^+]$ and $J_{ijk} = [X_{ijk}^-, X_{ijk}^+]$ are declared before the integral is read. This placement associates a physical two-chain with the finite face; it does not identify a graph triangle with a spacetime shape.

The value below is read in the gauge fixed by (33), together with $\chi_{\text{ax}} = 0$. The constant component $A_{X^1} = a_{\text{hol}}/\rho$ is therefore part of the charge definition.

Proposition 3 (axial face charge)

For (33) and (35),

$$\xi_{\text{ax}}^\flat = dX^1, \quad d\xi_{\text{ax}}^\flat = 0, \quad \iota_{\xi_{\text{ax}}} A = \frac{a_{\text{hol}}}{\rho}, \quad \star_g F = q(u) du \wedge dX^1. \quad (54)$$

Consequently the axial charge two-form and the addressed face value are

$$\mathcal{Q}_{\text{ax}}^N = -\frac{a_{\text{hol}}}{\mu_0 \rho} q(u) du \wedge dX^1, \quad (55)$$

$$\mathcal{W}_{ijk}^{\text{ax}} := \int_{\Sigma_{ijk}} \iota_{ijk}^* \mathcal{Q}_{\text{ax}}^{\text{N}} = -\frac{a_{\text{hol}}}{\mu_0 \rho} \Delta X_{ijk}^1 [f(u_{ijk}^+) - f(u_{ijk}^-)]. \quad (56)$$

For a full compact axial circuit, $\Delta X_{ijk}^1 = 2\pi\rho$, this reads

$$\mathcal{W}_{ijk}^{\text{ax}} = -\frac{2\pi a_{\text{hol}}}{\mu_0} [f(u_{ijk}^+) - f(u_{ijk}^-)]. \quad (57)$$

Proof. The gravitational term in (51) vanishes because $\xi_{\text{ax}}^b = dX^1$ is closed. The orientation (50) gives the Hodge read in (54). Substitution into (51) yields (55). Pulling (55) back along (53), then using $q = f'$, gives (56). The full-circle case gives (57). Since $d\mathcal{Q}_{\text{ax}}^{\text{N}} = 0$, the value is invariant under deformations of Σ_{ijk} that preserve its oriented boundary inside the declared pp-wave chart. $\square$

The charge is a covariant Einstein–Maxwell face quantity. It remains distinct from the finite seam residue h_{ijk} , the metric curvature, the electromagnetic curvature F , holonomy, and proper-time difference.

The B -preserving boost L_s imposes the profile condition

$$q(e^{-s}u)e^{-s} = q(u) \quad (58)$$

for every s . It forces $uq(u) = c_{\text{ax}}$. Hence, on either connected half-line $u > 0$ or $u < 0$, its exact pp-wave profile is

$$q(u) = \frac{c_{\text{ax}}}{u}, \quad f(u) = c_{\text{ax}} \log |u| + f_0. \quad (58a)$$

Equation (56) gives the Einstein–Maxwell axial face charge. When $\mathcal{H}$ and the electroweak connections carry the same X^1 -symmetry, the same covariant construction extends to the full metric–Higgs action. The boost profile is singular at $u = 0$, so it is defined on either half-line chart.

10. Numerical convergence

The numerical calculation uses a manufactured exact electrovac pp-wave. It extends the axially symmetric solution of (35) by a harmonic vacuum mode,

$$\begin{aligned} ds_\epsilon^2 &= -2dudv + (dX^1)^2 + dy^2 + H_\epsilon du^2, \\ H_\epsilon(u, X^1, y) &= -\kappa_{\text{EM}} q^2 y^2 + \epsilon_{\text{vac}} \cos(kX^1) \cosh(ky), \\ F &= q du \wedge dy, \quad k = \rho^{-1}. \end{aligned} \quad (59)$$

The circle condition $X^1 \sim X^1 + 2\pi\rho$ is preserved, and

$$(\partial_{X^1}^2 + \partial_y^2)[\cos(kX^1) \cosh(ky)] = 0. \quad (59a)$$

Thus the continuous Einstein–Maxwell source remains $G_{uu} = \kappa_{\text{EM}} q^2$. The harmonic mode supplies a nonzero finite-difference truncation residual while the continuous field remains an exact solution. The numerical test uses

$$\rho = \kappa_{\text{EM}} = q = k = 1, \quad \epsilon_{\text{vac}} = \frac{1}{3}. \quad (59b)$$

This manufactured solution is X^1 -dependent. The axial charge of §9 belongs to the separate X^1 -invariant solution (35); its face charge is not evaluated on (59).

The computation uses the chart (u, v, X^1, y) , periodic boundary conditions in X^1 , analytic Dirichlet data in y , and a second-order central stencil. For the selected residual $\mathcal{E}_{uu} = r$, tetrad projection gives

$$\mathcal{E}_{\hat{0}\hat{0}} = \mathcal{E}_{\hat{0}\hat{3}} = \mathcal{E}_{\hat{3}\hat{0}} = \mathcal{E}_{\hat{3}\hat{3}} = r, \quad \|\mathcal{E}\|_v = 2|r|. \quad (59c)$$

With the reference scale $E_{\text{scale}} = 2\kappa_{\text{EM}}q^2$, the test normalizes the residual by that same factor. At the sampled grid maxima, $\varepsilon_{\text{GR}}^{(N)} = |r|/(\kappa_{\text{EM}}q^2)$; hence in the normalized test it equals $\max|\mathcal{E}_{uu}|$. The normalized residual is

$$\varepsilon_{\text{GR}}^{(N)} = \frac{\|\mathcal{E}_{\hat{a}\hat{b}}\|_v}{\max\left(\|G_{\hat{a}\hat{b}}\|_v, \left\|\frac{8\pi G_N}{c^4}T_{\hat{a}\hat{b}}\right\|_v, E_{\text{scale}}\right)}. \quad (59d)$$

The refinement sequence is

Resolution N	$\varepsilon_{\text{GR}}^{(N)}$
128	$7.388546940788 \times 10^{-4}$
256	$1.892846791529 \times 10^{-4}$
512	$4.790319212382 \times 10^{-5}$

With

$$p_N = \log_2 \frac{\varepsilon_{\text{GR}}^{(N)}}{\varepsilon_{\text{GR}}^{(2N)}}, \quad (60)$$

the observed orders are

$$p_{128} = 1.964733023553, \quad p_{256} = 1.982363941813. \quad (61)$$

Both lie within 0.05 of the second-order rate. The coarse 32-cell level is omitted as pre-asymptotic. These values establish the expected convergence for the specified discretization of the manufactured exact field. They do not add a source term or alter the continuous Einstein–Maxwell solution.

11. Synthesis

$$\boxed{\begin{aligned} (\mathcal{D}, \mathcal{C}, B) &\longrightarrow (Q, L_s, \gamma) \longrightarrow (r, \tilde{\gamma}) \\ &\longrightarrow (K_8, h_{ijk}) \longrightarrow (g, A, F, \mathcal{H}, q_n) \\ &\longrightarrow \rho(q_n) \longrightarrow (\epsilon_n, \Pi_n, R_n). \end{aligned}} \quad (62)$$

Relation	Carrier	Read
Signed difference	B on $\mathbb{R}^2$	Null lines and preserved boost relation.
Return	$r : X \rightarrow Y$	Equal output in a declared reader.
Memory	Lift and occurrence record	Distinct addressed occurrences through return.
Cup	Constrained path and mirror	Bounded motion with inverse transport.
Compatibility	K_8 cochain	Triangle residual and global potential.
Field realization	Einstein–Maxwell pp-wave	Exact on-shell residual.
Higgs scope	$\mathcal{H}, g, T^{\mathcal{H}}$	Gauge-invariant scalar and metric relation.
Visual measurement	Eye/CIE protocol	Declared (X, Y, Z) coordinate.
Conservation	Noether current	Charge associated with declared symmetry.

Scope recognition is the maintained ability to state which carrier, reader, invariant, and residual each report uses. Reference gives the comparison; return makes recurrence readable; residual identifies the remaining work.

12. Result: Metric Minimum

The metric minimum is:

$$\boxed{\text{Metric Minimum} = (\text{reference, oriented rotation, return, carriage, residual}).} \quad (63)$$

Oriented rotation supplies a signed direction through

$$\Omega^2 = -I, \quad \omega(u, v) = u^\top \Omega v, \quad H_B \Omega H_B = \Omega^{-1}. \quad (64)$$

Within the declared reader ρ , endurance is typed by

$$\text{Endurance}_\rho := (\Omega, \Theta, R, I), \quad (65)$$

where Ω carries oriented rotation, Θ gives the timing address, R retains occurrence through advance, and I is the invariant relation carried through the turn.

The axial cup gives the mechanical realization; the clock gives the regular occurrence realization; $\mathcal{H}^\dagger \mathcal{H}$ gives the gauge-invariant field-scalar realization; the metric g carries the relational scope; the eye/CIE packet gives an embodied observer realization.

The reader reports completed local fit by

$$\Phi(\pi) = \text{Silence}_\rho \iff \epsilon_\rho = 0. \quad (66)$$

Silence is a zero unresolved residual in a declared comparison. Physical advance, field dynamics, and retained occurrence continue through that fit.

13. Historical and formal coordinates

The geometry and transport used here follow Riemann and Levi-Civita. The field equations are those of Maxwell and Einstein. The conserved-current construction uses Noether's variational theorem in the covariant form developed by Iyer and Wald. Brinkmann, Baldwin–Jeffery, and Peres provide the exact null-field coordinates. The visual reader follows the CIE standard observer functions; the modular and golden-field statements follow the standard references of Beardon and Hardy–Wright. The gauge-scalar sector follows Higgs and Weinberg.

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