COPPALINI ARCHITECTURE

Gravity Light

Metric invariance and fixed null directions

Anthony Vito Coppa

9 November 2026

Abstract

Gravity measures interval. Light propagates along the null relation of that interval. A recurrence matrix and an oriented area form generate a symmetric split metric with two fixed null directions. Its metric flow is a Lorentzian boost and its mirror reverses that boost. Every chosen Lorentzian two-plane of spacetime has this root as an explicit normal form. Einstein curvature and Maxwell curvature then describe the gravitational and electromagnetic fields on the spacetime carrier. The resulting equations relate metric, null motion, field strength, clock difference, and measured occurrence at one address.

1. Metric field

Physical interval is the Lorentzian quadratic form

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu. \tag{GL.0}$$

Gravity is the curvature relation of g. Light follows the null condition

$$ds^2 = 0. \tag{GL.0a}$$

At each spacetime address $x \in M$, a Lorentzian two-plane $P_x \subset T_xM$ carries one time direction, one transverse spatial direction, and the two null directions between them. The following finite root is an exact normal form of that local relation.

2. The split metric

Let

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad C = -\Omega,$$

$$G = [Q, \Omega] = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}, \qquad H = \frac{G}{\sqrt{5}}, \qquad S = \frac{2Q - I}{\sqrt{5}}.$$

The common exact identities are

$$Q^2 = Q + I, \qquad C^2 = -I, \qquad G^2 = 5I, \qquad H^2 = S^2 = I,$$

$$H = CS, \qquad SG = \sqrt{5}\Omega = -GS, \qquad L_s := e^{sS}, \qquad L_s^{\mathsf{T}} G L_s = G, \qquad H L_s H = L_{-s}.$$

Theorem 1 (Root identities)**.** *The matrices above satisfy*

$$G = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}, \qquad G^2 = 5I, \qquad \det G = -5, \qquad \operatorname{sig}(G) = (1,1). \tag{GL.1}$$

Moreover,

$$S^{\mathsf{T}}G + GS = 0, \qquad H^{\mathsf{T}}GH = G, \qquad HSH = -S. \tag{GL.2}$$

Proof. Direct multiplication gives $Q^2 = Q + I$, $C^2 = -I$, and $G = Q\Omega - \Omega Q$. Put $A = 2Q - I$. Then

$$A^2 = 5I, \qquad G^2 = 5I, \qquad AG = 5\Omega = -GA.$$

Dividing by $\sqrt{5}$ gives the first two relations in GL.2. Since H and S are symmetric involutions, the remaining identities follow directly. □

The metric flow is

$$L_s = e^{sS} = \cosh(s)I + \sinh(s)S. \tag{GL.3}$$

Differentiating $L_s^{\mathsf{T}}GL_s$ and using GL.2 yields

$$L_s^{\mathsf{T}}GL_s = G, \qquad HL_sH = L_{-s}. \tag{GL.4}$$

Thus the mirror reverses the flow while preserving the metric relation.

3. Null directions and fixed metric motion

The null condition for $z = (x, y)^{\mathsf{T}}$ is

$$z^{\mathsf{T}}Gz = -2x^2 + 2xy + 2y^2 = 0. \tag{GL.5}$$

Its two projective solutions are

$$\ell_\pm = \mathbb{R}\left(1, \frac{-1 \pm \sqrt{5}}{2}\right)^{\mathsf{T}}. \tag{GL.6}$$

Proposition 2 (Fixed null directions)**.** *Let*

$$A_G := G\Omega = -\sqrt{5}S, \qquad \mathsf{B}_t := e^{tA_G} = L_{-\sqrt{5}t}. \tag{GL.7}$$

Then

$$\mathsf{B}_t\ell_+ = e^{-\sqrt{5}t}\ell_+, \qquad \mathsf{B}_t\ell_- = e^{+\sqrt{5}t}\ell_-. \tag{GL.8}$$

Proof. The two lines are the eigendirections of S with opposite eigenvalues. Apply the exponential in GL.7. □

The two lines are the two null directions of the local split metric. The boost changes scale along them in opposite directions while leaving each direction fixed.

4. Local Lorentzian geometry

Define

$$A_{\mathrm{Lor}} := \begin{pmatrix} \sqrt{5/2} & 0 \\ 1/\sqrt{2} & \sqrt{2} \end{pmatrix}, \qquad \eta_{1,1} := \operatorname{diag}(-1, 1). \tag{GL.9}$$

A direct multiplication gives

$$A_{\mathrm{Lor}}^{\mathsf{T}}\eta_{1,1}A_{\mathrm{Lor}} = G. \tag{GL.10}$$

Consequently, with $\widetilde{A} = A_{\mathrm{Lor}} \oplus I_2$ and $\eta_{3,1} = \operatorname{diag}(-1, 1, 1, 1)$,

$$\widetilde{A}^{\mathsf{T}}\eta_{3,1}\widetilde{A} = G \oplus I_2. \tag{GL.11}$$

This is an exact congruence of bilinear forms. On a curved Lorentzian spacetime, choose at x an isometric inclusion

$$\iota_x : (\mathbb{R}^2, G) \longrightarrow (P_x, g_x), \qquad g_x(\iota_x u, \iota_x v) = u^{\mathsf{T}} G v. \tag{GL.11a}$$

The root null directions become physical local null directions:

$$g_x(\iota_x \ell_\pm, \iota_x \ell_\pm) = 0. \tag{GL.11b}$$

The root boost becomes the Lorentzian boost

$$\mathsf{B}_t^{(x)} = \iota_x \mathsf{B}_t \iota_x^{-1} \quad \text{on } P_x. \tag{GL.11c}$$

5. Sevenfold transverse rotation

Let $E = \mathbb{R}^2$ carry the Euclidean metric I_2 and the oriented rotation generator C. Define the four-dimensional carrier

$$\widetilde{V} = \mathbb{R}^2 \oplus E, \qquad \widetilde{G} = G \oplus I_2, \qquad \widetilde{C} = C \oplus C. \tag{GL.11d}$$

With $D = \mathrm{diag}(1, -1)$, the mirror on the extended carrier is

$$\widetilde{H} = H \oplus D. \tag{GL.11e}$$

The relation $DCD = -C$ combines with $HCH = -C$ to give

$$\widetilde{H}^{\mathsf{T}} \widetilde{G} \widetilde{H} = \widetilde{G}, \qquad \widetilde{H} \widetilde{C} \widetilde{H} = -\widetilde{C}, \qquad \widetilde{H}^2 = I_4. \tag{GL.11f}$$

For the cyclic generator τ of the seven-address transport, put

$$R_r := e^{(2\pi r/7)C}, \qquad \varrho(\tau^r) := I_2 \oplus R_r, \qquad r \in \mathbb{Z}_7. \tag{GL.11g}$$

Then ϱ is a faithful real representation of $\mathbb{Z}_7$ in the Lorentzian isometry group:

$$\varrho(\tau^r)\varrho(\tau^s) = \varrho(\tau^{r+s}), \qquad \varrho(\tau)^7 = I_4, \qquad \varrho(\tau)^{\mathsf{T}} \widetilde{G} \varrho(\tau) = \widetilde{G}. \tag{GL.11h}$$

It commutes with phase and reverses under the mirror:

$$\varrho(\tau)^{-1} \widetilde{C} \varrho(\tau) = \widetilde{C}, \qquad \widetilde{H} \varrho(\tau) \widetilde{H} = \varrho(\tau)^{-1}. \tag{GL.11i}$$

The embedded null directions are

$$\widetilde{\ell}_\pm = (\ell_\pm, 0) \in \widetilde{V}, \qquad \varrho(\tau^r)\widetilde{\ell}_\pm = \widetilde{\ell}_\pm. \tag{GL.11j}$$

The sevenfold transport rotates the transverse phase plane while leaving the two local gravity–light null directions fixed. The mirror exchanges their projective directions and reverses the cyclic rotation.

6. Gravity and light

The Einstein curvature tensor is

$$\mathrm{Ein}_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{GL.12}$$

The gravitational field equation is

$$\mathrm{Ein}_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}. \tag{GL.12a}$$

For an electromagnetic two-form F, Maxwell curvature obeys

$$dF = 0, \qquad d \star_g F = J. \tag{GL.13}$$

In vacuum $J = 0$. In geometric optics, a wave covector k is null when

$$g^{-1}(k, k) = 0. \tag{GL.14}$$

A plane-fronted gravity–light field has the form

$$ds^2 = 2\, du\, dv + \mathcal{H}_{\mathrm{pp}}(u, x, y)\, du^2 + dx^2 + dy^2, \qquad R_{uu} = -\frac{1}{2} \Delta_\perp \mathcal{H}_{\mathrm{pp}}. \tag{GL.15}$$

In vacuum, $R_{uu} = 0$ gives $\Delta_\perp \mathcal{H}_{\mathrm{pp}} = 0$. Thus curvature, null propagation, and the local split metric meet in one spacetime field.

7. Direct observation

A physical field has an interface

$$\begin{aligned}\mathfrak{I}_J := (J, \Gamma_J, \mathrm{Src}_J; A_J, X_J, Y_J, T_J, \Sigma_J, \mathrm{Con}_J,\\ \rho_J, O_J, K_J, B_J, \theta_J), \qquad \rho_J : X_J \to Y_J, \quad O_J : X_J \to \mathcal{O}_J, \quad \theta_J : X_J \to \{0,1\}.\end{aligned} \tag{GL.16}$$

Its occurrence type is

$$\begin{aligned}\mathsf{Occ}_J := \{(\mathfrak{I}_J; \Theta, x, \rho_J(x), \kappa, \boldsymbol{\varepsilon}, \theta_J(x)) :\\ \Theta \in A_J,\ x \in X_J,\ \kappa \in K_J,\ \boldsymbol{\varepsilon} \in \mathbb{R}^3_{\geq 0}\}.\end{aligned} \tag{GL.17}$$

An indexed occurrence $\mathsf{Occ}_{J,n}$ is an element of Occ_J.

For $J = \mathsf{GL}$, take the state coordinates of x_n to include the local metric g_n, clock difference $\Delta\tau_n$, wave covector k_n, and electromagnetic two-form F_n. With $O_n = O_{\mathsf{GL}}(x_n)$, the gravity–light projection is

$$\mathrm{pr}_{\mathrm{GL}}(\mathsf{Occ}_{\mathsf{GL},n}) = (\Theta_n, g_n, \Delta\tau_n, k_n, F_n, O_n, \kappa_n, \boldsymbol{\varepsilon}_n). \tag{GL.18}$$

The physical relation is read by comparing these quantities at their common address.