COPPALINI ARCHITECTURE

Time Scope

Local phase and projective return

Anthony Vito Coppa

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Abstract

Time carries local phase through physical distance and duration. Scope reads the direction of a state through its projective identity. A half-turn changes every nonzero local state while leaving its projective scope unchanged. The mirror reverses phase. Linear polarization and a two-level quantum state carry the same double-angle relation: local phase advances by α while the scope read advances by 2α. The field therefore separates local change, projective return, measurement direction, and time-addressed motion.

1. Time and scope

For a wave of angular frequency ω and wave covector $\mathbf{k}$, phase difference across a spacetime interval is

$$\Delta\alpha = \omega\,\Delta t - \mathbf{k}\cdot\Delta\mathbf{x}. \tag{TS.0}$$

For light in vacuum,

$$\omega = c|\mathbf{k}|, \qquad \lambda_{\text{wave}} = \frac{2\pi}{|\mathbf{k}|}, \qquad c = \lambda_{\text{wave}} f. \tag{TS.0a}$$

For a material trajectory with velocity $\mathbf{v}$,

$$\Delta\mathbf{x} = \mathbf{v}\,\Delta t, \qquad \Delta\alpha = (\omega - \mathbf{k}\cdot\mathbf{v})\Delta t. \tag{TS.0b}$$

Thus duration, distance, wavelength, frequency, and light speed determine one physical phase relation. The local phase carrier is $V^\times = \mathbb{R}^2 \setminus \{0\}$. Scope reads the unoriented line through its state.

The half-turn relation is

$$\operatorname{sc}(U_\pi z) = \operatorname{sc}(z), \qquad U_\pi z \neq z \quad (z \in V^\times). \tag{TS.1}$$

It is a return of scope through local difference.

2. Phase carrier and mirror

Let

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad C = -\Omega,$$

$$G = [Q, \Omega] = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}, \qquad H = \frac{G}{\sqrt{5}}, \qquad S = \frac{2Q - I}{\sqrt{5}}.$$

The common exact identities are

$$Q^2 = Q + I, \qquad C^2 = -I, \qquad G^2 = 5I, \qquad H^2 = S^2 = I,$$
$$H = CS, \qquad SG = \sqrt{5}\Omega = -GS, \qquad L_s := e^{sS}, \qquad L_s^{\mathsf{T}} G L_s = G, \qquad HL_sH = L_{-s}.$$

Define the phase transport

$$U_\alpha = e^{\alpha C} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \qquad \alpha \in \mathbb{R}/2\pi\mathbb{Z}. \tag{TS.2}$$

Proposition 1 (Phase and mirror)**.** *For all α,*

$$U_{\alpha+\beta} = U_\alpha U_\beta, \qquad HU_\alpha H = U_{-\alpha}, \qquad U_\pi = -I.$$

Proof. The composition law is the one-parameter exponential law. Since $HC = -CH$, $He^{\alpha C}H = e^{-\alpha C}$. Finally $e^{\pi C} = I\cos\pi + C\sin\pi = -I$. □

The split form supplies two null lines

$$\ell_\pm = \mathbb{R}\left(1, \frac{-1 \pm \sqrt{5}}{2}\right)^{\mathsf{T}}, \qquad \ell_\pm^{\mathsf{T}} G \ell_\pm = 0. \tag{TS.3}$$

The mirror and quarter-turn exchange these two projective directions.

3. Scope and the half-turn receipt

Definition 2 (Scope reader)**.** The scope reader is the projectivization

$$\mathrm{sc} : V^\times \longrightarrow \mathbb{R}P^1, \qquad \mathrm{sc}(z) = [z]. \tag{TS.4}$$

It forgets nonzero scalar magnitude and sign while preserving the line through the origin.

Define the scope-closure selector

$$\Phi : \mathbb{R}/2\pi\mathbb{Z} \longrightarrow \mathcal{P}(V^\times), \qquad \Phi(\alpha) := \{z \in V^\times : \mathrm{sc}(U_\alpha z) = \mathrm{sc}(z)\}, \qquad \mathsf{Silence}_{\mathrm{scope}} := V^\times. \tag{TS.4a}$$

Theorem 3 (Half-turn scope return)**.** *For every $z \in V^\times$,*

$$U_\pi z = -z \neq z, \qquad \mathrm{sc}(U_\pi z) = \mathrm{sc}(z). \tag{TS.5}$$

Thus the half-turn receipt is

$$\Phi(\pi) = \mathsf{Silence}_{\mathrm{scope}}. \tag{TS.6}$$

Proof. The first equality is the preceding proposition. A nonzero vector and its negative span the same one-dimensional real subspace, so their projective readings are equal. A nonzero real vector cannot equal its negative. □

The projective action of H is well typed:

$$\overline{H} : \mathbb{R}P^1 \to \mathbb{R}P^1, \qquad \overline{H}([z]) = [Hz]. \tag{TS.7}$$

It is a bijection and supplies scope covariance,

$$\mathrm{sc}(Hz) = \overline{H}(\mathrm{sc}(z)). \tag{TS.8}$$

For an explicit circle coordinate, define

$$\chi([x : y]) = \left(\frac{x^2 - y^2}{x^2 + y^2}, \frac{2xy}{x^2 + y^2}\right) \in S^1. \tag{TS.9}$$

If $z = (\cos\alpha, \sin\alpha)$, then

$$\chi(\mathrm{sc}(z)) = (\cos 2\alpha, \sin 2\alpha). \tag{TS.10}$$

The doubled angle is the scope coordinate of the two-to-one phase relation.

4. Ordered transport and memory return

Let $(A, \preceq)$ be an address order with initial address a_0. Let $\phi : A_{\succeq a_0} \to \mathbb{R}/2\pi\mathbb{Z}$ be its phase coordinate. Define

$$W_{b\leftarrow a} := U_{\phi(b)-\phi(a)}, \qquad a \preceq b. \tag{TS.11}$$

Proposition 4 (Functorial transport)**.** *For $a \preceq b \preceq c$,*

$$W_{c\leftarrow b}W_{b\leftarrow a} = W_{c\leftarrow a}, \qquad W_{a\leftarrow a} = I. \tag{TS.12}$$

For $\psi_0 \in V^\times$, the trajectory is

$$\Psi_{a_0,\psi_0}(a) := W_{a\leftarrow a_0}\psi_0, \tag{TS.13}$$

and its trajectory-relative displacement is

$$\delta_{\mathrm{traj}}(a) := \Psi_{a_0,\psi_0}(a) - \psi_0. \tag{TS.14}$$

The displacement is measured from the initial state at a_0.

A physical field has an interface

$$\begin{aligned}\mathfrak{I}_J := (&J, \Gamma_J, \mathrm{Src}_J; A_J, X_J, Y_J, T_J, \Sigma_J, \mathrm{Con}_J,\\ &\rho_J, O_J, K_J, B_J, \theta_J), \qquad \rho_J : X_J \to Y_J, \quad O_J : X_J \to \mathcal{O}_J, \quad \theta_J : X_J \to \{0,1\}.\end{aligned} \tag{TS.14a}$$

Its occurrence type is

$$\begin{aligned}\mathsf{Occ}_J := \{&(\mathfrak{I}_J; \Theta, x, \rho_J(x), \kappa, \boldsymbol{\varepsilon}, \theta_J(x)) :\\ &\Theta \in A_J,\ x \in X_J,\ \kappa \in K_J,\ \boldsymbol{\varepsilon} \in \mathbb{R}^3_{\geq 0}\}.\end{aligned} \tag{TS.14b}$$

An indexed occurrence $\mathsf{Occ}_{J,n}$ is an element of Occ_J.

For $J = \mathsf{TS}$, take $\Theta_n = a_n$, $x_n = \Psi_{a_0,\psi_0}(a_n)$, and $y_n = \mathrm{sc}(x_n)$. The time–scope projection is

$$\mathrm{pr}_{\mathrm{TS}}(\mathsf{Occ}_{\mathsf{TS},n}) = (\Theta_n, x_n, y_n, \delta_{\mathrm{traj}}(a_n), \boldsymbol{\varepsilon}_n). \tag{TS.14c}$$

Definition 5 (Memory return)**.** For a reader $\rho : V^\times \to Y$, define

$$\mathrm{MemRet}_\rho(a, b; \psi_0) \iff \begin{cases} a \prec b, \\ \rho(\Psi(a)) = \rho(\Psi(b)), \\ \exists m\ (a \prec m \prec b\ \wedge\ \rho(\Psi(m)) \neq \rho(\Psi(a))). \end{cases} \tag{TS.15}$$

The intermediate address witnesses departure between the equal endpoint reads. For the scope reader, a half-turn with an intermediate quarter-turn gives memory return:

$$\phi(b) - \phi(a) = \pi, \qquad \phi(m) - \phi(a) = \frac{\pi}{2} \quad \Longrightarrow \quad \mathrm{MemRet}_{\mathrm{sc}}(a, b; \psi_0). \tag{TS.16}$$

5. Mirror transfer

Define mirrored phase data by $\phi^\mu = -\phi$ and mirrored trajectory by

$$\Psi^\mu(a) := H\Psi(a). \tag{TS.17}$$

Then Ψ^μ has transport $W^\mu_{b\leftarrow a} = U_{-(\phi(b)-\phi(a))}$.

Theorem 6 (Mirror transfer of memory return)**.** *Suppose a reader ρ has an injective mirror covariance*

$$\rho(Hz) = \overline{H}_\rho(\rho(z)) \tag{TS.18}$$

for an injective map $\overline{H}_\rho$ on its image. Then

$$\mathrm{MemRet}_\rho(a, b; \psi_0) \iff \mathrm{MemRet}_\rho(a, b; H\psi_0) \text{ on mirrored data.} \tag{TS.19}$$

Proof. The mirror relation gives $\Psi^\mu(x) = H\Psi(x)$ at every address x. Covariance transfers endpoint equality. Injectivity transfers the strict inequality at the intermediate departure address. The converse follows because $H^2 = I$. □

The scope reader satisfies the hypothesis by TS.8, because $\overline{H}$ is a projective bijection.

6. Light and two-level matter

6.1. Linear polarization

Choose a transverse real electric-field amplitude $E \in \mathbb{R}^2 \setminus \{0\}$ at a detector plane. Its unoriented polarization axis is $[E] \in \mathbb{R}P^1$. A polarizer reads the scope direction:

$$E \longmapsto [E]. \tag{TS.20}$$

The half-turn $E \mapsto -E$ preserves the polarization axis. Wavelength, detector plane, polarizer convention, intensity threshold, and calibration fix the measured read.

6.2. Two-level realization

Let $\sigma_x, \sigma_y, \sigma_z$ be the Pauli matrices and define

$$V_\alpha = e^{-i\alpha\sigma_z}. \tag{TS.21}$$

For $|+x\rangle$, the Bloch read is

$$\big(\langle\sigma_x\rangle, \langle\sigma_y\rangle\big) = (\cos 2\alpha, \sin 2\alpha), \qquad V_{\alpha+\pi} = -V_\alpha. \tag{TS.22}$$

The Bloch vector reads the double-cover geometry directly: state phase carries the local turn and the measured transverse direction carries the scope turn.