Minimum Endurance Delta

Dual-Form Motion, Cup Symmetry, and the Positive Horizon of Return

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Abstract

We construct one self-contained classical model in which a two-dimensional carrier supports an alternating difference form, a derived symmetric interval, Fibonacci transport, mirror inversion, finite square phase, infinite-dihedral addressed motion, a bounded-curvature cup, Noether conservation, axial rotation, spinorial double return, a twenty-eight-difference occurrence clock, a positive unattained projective horizon, and an explicit source-free light-flow witness. The algebraic hinge is the ordered pair

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

which generates the symmetric form $G = Q^{\mathsf{T}}\Omega - \Omega Q$ rather than requiring a second primitive reader. One Q-step reverses both the alternating and symmetric forms; two steps restore them. The two null rays of G are the golden directions Φ and Ψ, whose oriented two-form separation is exactly $\sqrt{5}$. A normalized reflection exchanges the rays and conjugates projective Fibonacci motion to its inverse, yielding D_∞ with a four-phase visible quotient. A general Noether identity is then proved from the variational hypothesis and specialized to axial turn symmetry, where angular momentum is the conserved rotational current of a cup-invariant motion. Spin and the occurrence clock show that return of a displayed state need not identify the lift or erase the passage. Finally, Fibonacci convergence supplies a positive horizon never occupied by a finite rational iterate; natural-number successor supplies the corresponding local form called Tomorrow; and an explicit Lorentzian $SO(2)$ Maxwell field supplies a null line-flow realization. The main result is a compatibility theorem: these structures inhabit one exact construction while their carriers, values, histories, and physical offices remain non-coercible.

Keywords: dual forms; Fibonacci matrix; cup symmetry; Noether theorem; angular momentum; spin; infinite dihedral group; non-erasing return; twenty-eight differences; null flow; horizon.

Jurisdiction: ordinary classical linear algebra, projective and hyperbolic geometry, finite combinatorics, calculus of variations, covering spaces, Lorentzian geometry, and source-free Abelian gauge theory. Every physical name below is attached only after its mathematical carrier has been declared.

I. Statement and scope

The paper asks one question: what exact structure is sufficient for ordered motion to reverse, return, remain distinguishable from rest, and continue without losing its address? The answer is given by one explicit tuple rather than by a verbal axiom.

Theorem 1 (Minimum Endurance Delta). *There exists a tuple*

$$\mathfrak{E}_\Delta = (V, \omega, Q, G, H_{\sqrt{5}}, P_4, u, E_8, \Omega_8, \mathsf{Scan}_{28}, \mathsf{MR}, \mathfrak{C}, L, J_X, U_{1/2}, M, g, E, A_a, F)$$

carrying simultaneously:

(i) *a nonzero alternating form ω and a symmetric form g_Q generated from the same Fibonacci carriage Q;*

(ii) *one-step simultaneous sign reversal and two-step dual-form return;*

(iii) *a closed four-dimensional operator algebra* $\operatorname{span}\{I, Q, \Omega, G\}$;

(iv) *two golden null rays whose oriented ζ_ω separation is $\sqrt{5}$;*

(v) *a Euclidean mirror $H_{\sqrt{5}}$ and an infinite-dihedral projective action with a four-phase square quotient;*

(vi) *a typed Memory Return datum separating base return, lift, represented difference, and receipt;*

(vii) *a geometric cup with bounded curvature and a return reader;*

(viii) *the general Noether charge for a one-parameter variational symmetry;*

(ix) *axial turn symmetry with conserved angular momentum, including exact central-force orbit specialization;*

(x) *the $SU(2) \to SO(3)$ spin double cover with a non-silent 2π return and silent 4π represented return;*

(xi) *a non-erasing four-phase clock, two-lift eight-address window, all twenty-eight pairwise seams, and an explicit seven-by-four scanner;*

(xii) *a positive projective horizon Φ approached but never occupied by a finite Fibonacci rational iterate;*

(xiii) *a successor operator with no fixed point or terminal natural-number address, giving the exact local office called Tomorrow; and*

(xiv) *an explicit nonzero source-free Maxwell plane-wave curvature on a Lorentzian manifold, with null propagation and independently tunable global holonomy.*

No equality is asserted among a scalar two-form read, a group element, a projective point, an occurrence address, a Noether charge, a receipt, a null line, a Maxwell curvature, or a horizon. Their coexistence is proved by construction; their non-coercion is part of the theorem.

The proof occupies Sections II–XIII. Section XIV records the strict boundary; Section XV gives a single-page definition register.

II. The finite dual-form kernel

Let $V = \mathbb{R}^2$ with its standard Euclidean inner product. Define

$$Q := \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \omega(u, v) := u^{\mathsf{T}} \Omega v.$$

The matrix Ω is simultaneously the coordinate matrix of the standard oriented area form and, after Euclidean index-raising, the quarter-turn operator. These offices share a matrix but not a tensor type.

Proposition 2 (Fibonacci carriage). *The matrix Q satisfies*

$$Q^2 = Q + I, \qquad Q^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix} \quad (n \geq 1),$$

where $F_0 = 0$, $F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$.

Proof. Cayley–Hamilton gives $Q^2 - Q - I = 0$. The power formula follows by induction: if $Q^n = F_n Q + F_{n-1} I$, then $Q^{n+1} = F_n(Q + I) + F_{n-1} Q = F_{n+1} Q + F_n I$. □

This is the classical Fibonacci Q-matrix realization [4].

Define the symmetric bilinear form

$$g_Q(u, v) := \omega(Qu, v) - \omega(u, Qv).$$

Its matrix is not primitive:

$$G := Q^{\mathsf{T}} \Omega - \Omega Q = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}.$$

Since $\Omega^{\mathsf{T}} = -\Omega$, $G^{\mathsf{T}} = G$.

Theorem 3 (Dual-form sign reversal). *For all $u, v \in V$,*

$$\omega(Qu, Qv) = -\omega(u, v), \qquad g_Q(Qu, Qv) = -g_Q(u, v).$$

Consequently

$$(Q^2)^*\omega = \omega, \qquad (Q^2)^* g_Q = g_Q.$$

Proof. Direct multiplication gives

$$Q^{\mathsf{T}}\Omega Q = -\Omega, \qquad Q^{\mathsf{T}} G Q = -G.$$

Applying the same congruence twice restores both forms. Equivalently, the second identity follows without coordinates from $Q^2 = Q + I$:

$$\begin{aligned} g_Q(Qu, Qv) &= \omega(Q^2u, Qv) - \omega(Qu, Q^2v) \\ &= \omega((Q + I)u, Qv) - \omega(Qu, (Q + I)v) \\ &= \omega(u, Qv) - \omega(Qu, v) = -g_Q(u, v). \end{aligned}$$

□

Definition 4 (Dual-form return). The operator

$$\mathcal{R}_Q := Q^2$$

is the *dual-form return* of the kernel. The name refers only to the pullback identities in Theorem 3; it does not assert $Q^2 v = v$.

The alternating unit zero is therefore relational:

$$\boxed{\omega + Q^*\omega = 0}.$$

It does not assert $\omega = 0$; the two nonzero oriented reads cancel.

Proposition 5 (Closed operator algebra). *With $[A, B] = AB - BA$ and $\{A, B\} = AB + BA$,*

$$\boxed{\begin{aligned} \{Q, \Omega\} &= \Omega, \quad [Q, \Omega] = G, \\ \{Q, G\} &= G, \quad [Q, G] = 5\Omega. \end{aligned}}$$

Hence the algebra generated by Q and Ω closes in $\operatorname{span}\{I, Q, \Omega, G\}$*, and*

$$\operatorname{ad}_Q^2(\Omega) = [Q, [Q, \Omega]] = 5\Omega.$$

Proof. All four identities follow by direct multiplication. Since $Q^2 = Q + I$, products of higher degree reduce to the displayed span. □

Thus difference is not appended after motion. In this kernel the symmetric difference G is the exact residue of applying carriage and orientation in opposite orders.

III. Golden difference and the light-line normal form

For $v = (x, y)^{\mathsf{T}}$ define

$$\Delta_G(v) := v^{\mathsf{T}} G v = -2(x^2 - xy - y^2).$$

The eigenvalues of Q are

$$\Phi := \frac{1 + \sqrt{5}}{2}, \qquad \Psi := \frac{1 - \sqrt{5}}{2} = -\Phi^{-1},$$

with

$$\Phi + \Psi = 1, \qquad \Phi\Psi = -1, \qquad \Phi - \Psi = \sqrt{5}.$$

Let

$$v_\Phi := \begin{pmatrix} \Phi \\ 1 \end{pmatrix}, \qquad v_\Psi := \begin{pmatrix} \Psi \\ 1 \end{pmatrix}, \qquad P := \begin{pmatrix} \Phi & \Psi \\ 1 & 1 \end{pmatrix}.$$

Theorem 6 (Golden seam). *The projective null set of G is exactly*

$$\Delta_G(v) = 0 \quad \Longleftrightarrow \quad [v] \in \{[v_\Phi], [v_\Psi]\}.$$

Moreover

$$P^{-1}QP = \operatorname{diag}(\Phi, \Psi), \qquad P^{\mathsf{T}}GP = 5\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad P^{\mathsf{T}}\Omega P = \sqrt{5}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

In particular,

$$\boxed{\zeta_\omega(v_\Phi, v_\Psi) = \omega(v_\Phi, v_\Psi) = \sqrt{5}.}$$

Proof. The null equation for $y \neq 0$ is $(x/y)^2 - (x/y) - 1 = 0$, giving Φ and Ψ; neither coordinate axis is an additional null ray. The three matrix identities follow by substitution. The last equality is the $(1, 2)$ entry of $P^{\mathsf{T}}\Omega P$. □

Remark 7 (Resolution notation). The display

$$\boxed{\Phi\{+\zeta+\}\Psi = \sqrt{5}}$$

is used only as typed shorthand for $\zeta_\omega(v_\Phi, v_\Psi) = \sqrt{5}$. Its scalar shadow is $\Phi - \Psi = \sqrt{5}$. The braces do not introduce a second addition law or a scalar inverse of ζ_ω.

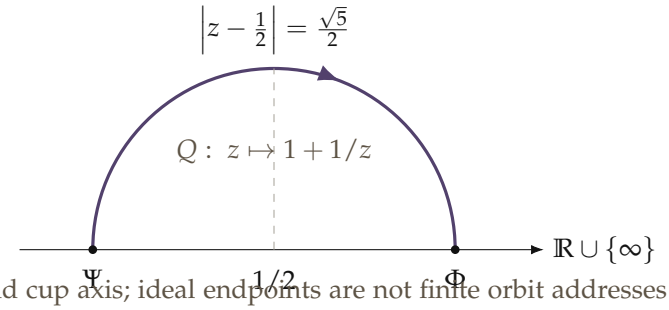
In the golden basis, the quadratic form is

$$5(a, b)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 10ab.$$

Under the linear change

$$t = \frac{a+b}{\sqrt{2}}, \qquad x = \frac{a-b}{\sqrt{2}},$$

one has $2ab = t^2 - x^2$. Thus the two golden eigendirections become the two null coordinate lines of a $(1, 1)$ Lorentzian plane. We name $\ell_\Phi = \mathbb{R}v_\Phi$ the *lift line* and $\ell_\Psi = \mathbb{R}v_\Psi$ the *return line*. These are names on an exact linear normal form; no numerical identification of $\sqrt{5}$ with a physical light speed is made.



mirror-held cup axis; ideal endpoints are not finite orbit addresses

Figure 1: The hyperbolic cup of the Fibonacci carriage in centered coordinates. The two golden rays are its ideal boundary; finite rational iterates remain on the carried orbit.

IV. Mirror wheel and infinite-dihedral closure

Normalize the symmetric form by

$$H_{\sqrt{5}} := \frac{G}{\sqrt{5}}, \qquad S_Q := \frac{2Q - I}{\sqrt{5}}.$$

Theorem 8 (Two reflections and one wheel). *The matrices $H_{\sqrt{5}}$ and S_Q are Euclidean reflections:*

$$H_{\sqrt{5}}^2 = S_Q^2 = I, \qquad H_{\sqrt{5}}^{\mathsf{T}} H_{\sqrt{5}} = S_Q^{\mathsf{T}} S_Q = I, \qquad \det H_{\sqrt{5}} = \det S_Q = -1.$$

Their ordered products are

$$\boxed{S_Q H_{\sqrt{5}} = \Omega}, \qquad \boxed{H_{\sqrt{5}} S_Q = \Omega^{-1} = -\Omega}.$$

Consequently

$$H_{\sqrt{5}} \Omega H_{\sqrt{5}} = \Omega^{-1}, \qquad \langle \Omega, H_{\sqrt{5}} \rangle \cong \mathrm{Dih}_4,$$

where Dih_4 *denotes the order-eight symmetry group of the square.*

Proof. Use $G^2 = 5I$, $(2Q - I)^2 = 5I$, $2Q - I = \Omega G$, and $G\Omega = -\Omega G$. The determinant statements follow directly. □

The same mirror also reverses Fibonacci carriage:

$$H_{\sqrt{5}} Q H_{\sqrt{5}} = -Q^{-1}.$$

Passing to projective classes $q = [Q]$ and $h = [H_{\sqrt{5}}]$ removes the scalar sign.

Theorem 9 (Infinite-dihedral Fibonacci motion). *The projective classes satisfy*

$$h^2 = 1, \qquad hqh = q^{-1}.$$

The element q has infinite order, and therefore

$$\boxed{\langle q, h \mid h^2 = 1,\ hqh = q^{-1} \rangle \cong D_\infty.}$$

Proof. The ratio of the eigenvalues of Q is $\Phi/\Psi = -\Phi^2$, not a root of unity, so q has infinite order. An involution conjugating an infinite-order generator to its inverse gives the standard infinite-dihedral presentation [5]. □

Define the visible phase representation by

$$q \longmapsto i, \qquad h \longmapsto K, \qquad K(z) = \bar{z}.$$

Since $KiK = i^{-1}$, this is a surjection $D_\infty \twoheadrightarrow \mathrm{Dih}_4$. The finite wheel closes orientation; the infinite lift retains the integer address.

V. MEMORY–ζ–MOTION

The finite reader is

$$\zeta_\omega : V \times V \to \mathbb{R}, \qquad \zeta_\omega(u, v) := \omega(u, v).$$

It is alternating and generally non-injective:

$$\zeta_\omega(v, u) = -\zeta_\omega(u, v), \qquad \zeta_\omega(v, v) = 0.$$

No scalar notation $\zeta_\omega^{-1}(d)$ is licensed. Inversion belongs to an address or transport carrying enough data to recover orientation.

Definition 10 (Return reader). Let X be a state space and $r : X \to Y$ a declared reader. A path $x : [0, 1] \to X$ is an r-return if

$$r(x(1)) = r(x(0)).$$

The equality is in Y only; it does not imply $x(1) = x(0)$ in X.

Definition 11 (Representation difference). For a representation $\rho : \Gamma \to \mathrm{GL}(W)$ define

$$\mathfrak{D}_\rho(g) := \rho(g) - I_W \in \mathrm{End}(W).$$

The letter Ψ remains reserved for the golden conjugate root.

Proposition 12 (Difference composition). *For $g_1, g_2 \in \Gamma$,*

$$\boxed{\mathfrak{D}_\rho(g_1 g_2) = \mathfrak{D}_\rho(g_1) + \mathfrak{D}_\rho(g_2) + \mathfrak{D}_\rho(g_1)\mathfrak{D}_\rho(g_2).}$$

Moreover

$$\mathfrak{D}_\rho(g) = 0 \quad \Longleftrightarrow \quad g \in \ker \rho.$$

Proof. Expand $\rho(g_1 g_2) - I = (I + \mathfrak{D}_\rho(g_1))(I + \mathfrak{D}_\rho(g_2)) - I$. The second statement is the definition of the kernel. □

Let $p : \widetilde{B} \to B$ be a declared covering or lift system with deck group Γ, let γ be a based loop in B, and let $\widetilde{\gamma}$ be a chosen lift. Write $\varepsilon \in \Gamma$ for the unique declared endpoint defect, and let $\rho : \Gamma \to \mathrm{GL}(W)$ be a representation.

Definition 13 (Memory Return datum). A Memory Return datum is a tuple

$$\mathsf{MR} = (r, x, p, \gamma, \widetilde{\gamma}, \varepsilon, \eta_{\mathrm{act}}, \rho, \mathfrak{D}_\rho(\varepsilon), \mathcal{R})$$

containing:

(a) a certified r-return x;

(b) the declared lift system p, based loop γ, and chosen lift $\widetilde{\gamma}$;

(c) an endpoint witness $\widetilde{\gamma}(1) = \varepsilon\, \widetilde{\gamma}(0)$;

(d) an action witness relating the loop to the return path;

(e) the represented difference $\mathfrak{D}_\rho(\varepsilon)$; and

(f) a receipt $\mathcal{R}$ retaining the path, lift, endpoint defect, representation read, evidence, and occurrence address.

No field is inferred solely from another field having the same numerical value.

The dual-form return Q^2 supplies one algebraic field of such a datum; an append-only receipt supplies a different field. Neither is renamed as the other.

Let an occurrence be $o_n = (u, n)$ for an opaque run identifier u, and let

$$\mathcal{H}_n = (r_0, \ldots, r_{n-1}).$$

Append is split:

$$\mathrm{dropLast}(\mathcal{H}_n \frown r_n) = \mathcal{H}_n, \qquad \mathrm{last}(\mathcal{H}_n \frown r_n) = r_n.$$

Thus equal read values at distinct occurrence indices remain distinct events.

Theorem 14 (Non-erasing dual return). *Let $v_{n+1} = Q v_n$. Then*

$$(-1)^n \Delta_G(v_n) = \Delta_G(v_0)$$

for every n. After two moves the complete dual-form relation returns,

$$(Q^2)^* \omega = \omega, \qquad (Q^2)^* g_Q = g_Q,$$

while generally $Q^2 v_n \neq v_n$. If a receipt is appended at each move, the later state has a strictly longer history even when a declared reader has returned.

Proof. The staggered invariant follows from $\Delta_G(Qv) = -\Delta_G(v)$. The pullback identities are Theorem 3. Split append preserves the entire prefix and last receipt independently of their read values. □

This is the minimum content of the phrase *Memory–ζ–Motion*: motion supplies the excursion, ζ_ω makes oriented difference readable, dual-form return restores the comparison law, and the receipt prevents silent return from becoming a blank event.

VI. Cup symmetry

Let (M, g_R) be a Riemannian configuration manifold, $I = [a, b]$, and $\gamma : I \to M$ a regular C^2 curve. Its covariant curvature magnitude is

$$\kappa_\gamma(t) := \frac{\| \operatorname{pr}_{\dot\gamma(t)^\perp} \nabla_{\dot\gamma}\dot\gamma \|_{g_R}}{\|\dot\gamma(t)\|_{g_R}^2}.$$

For a plane curve this becomes

$$\kappa_\gamma(t) = \frac{|\det(\dot\gamma(t), \ddot\gamma(t))|}{\|\dot\gamma(t)\|^3}.$$

Definition 15 (Cup datum). A geometric cup is a tuple

$$\mathfrak{C} = (M, g_R, C, I, \gamma, \kappa_{\max}, r, \eta_{\text{return}})$$

where $C \subseteq M$ is a declared constraint region, $\gamma(I) \subseteq C$,

$$0 \le \kappa_\gamma(t) \le \kappa_{\max} < \infty,$$

$r : C \to Y$ is a return reader, and

$$\eta_{\text{return}} : r(\gamma(b)) = r(\gamma(a)).$$

The cup is the bounded motion relation, not a claim that the image is a literal vessel.

Definition 16 (Mirror cup). A cup carries mirror symmetry when an involution $K_C : C \to C$ satisfies

$$K_C^2 = \mathrm{Id}_C, \qquad K_C \zeta_C K_C = \zeta_C^{-1}$$

for its admitted invertible transport $\zeta_C : C \to C$. Define the addressed return composite

$$\mathrm{Rem}_{\zeta_C}(x) := (\zeta_C^{-1} \circ \zeta_C)(x).$$

Its exact closure certificate is the equality

$$\mathrm{Rem}_{\zeta_C}(x) = x.$$

If a linear read chart $\chi : C \to W$ is separately supplied, the associated value residue is $\chi(\mathrm{Rem}_{\zeta_C}(x)) - \chi(x) = 0_W$.

Proposition 17 (Cup Möbius closure). *If ζ_C has infinite order, then*

$$\langle \zeta_C, K_C \mid K_C^2 = I, K_C \zeta_C K_C = \zeta_C^{-1} \rangle \cong D_\infty.$$

The inverse transport is the reverse orientation of the same office, not a second mechanism.

The finite kernel is an explicit mirror-return cup at the operator level: G supplies the signed inside/outside read, $H_{\sqrt{5}}$ supplies the mirror, $q = [Q]$ supplies transport, and Theorem 9 supplies exact Möbius closure. A geometric cup additionally requires the regular path, constraint region, and curvature bound declared above.

VII. Noether general form: symmetry of cup

The following is the standard finite continuous-symmetry form of Noether's first theorem, written with its boundary term explicit [1, 2]. Let $\mathcal{Q}$ be a smooth configuration manifold with local coordinates $q = (q^1, \dots, q^m)$, and let

$$S[\gamma] = \int_{t_0}^{t_1} L(t, q, \dot q)\, \mathrm{d}t$$

be a first-order action. Put

$$p_i := \frac{\partial L}{\partial \dot q^i}, \qquad H := p_i \dot q^i - L.$$

Consider an infinitesimal one-parameter transformation

$$\delta t = \varepsilon \tau(t, q), \qquad \delta q^i = \varepsilon \xi^i(t, q).$$

Theorem 18 (Noether charge with boundary term). *Assume there exists a smooth function $B(t,q)$ such that the prolonged variation satisfies*

$$L_t\tau + L_{q^i}\xi^i + p_i(D_t\xi^i - \dot{q}^i D_t\tau) + LD_t\tau = D_t B.$$

Then every Euler–Lagrange trajectory satisfies

$$\boxed{D_t J_X = 0}, \qquad \boxed{J_X = p_i\xi^i - H\tau - B}.$$

Proof. Along an Euler–Lagrange trajectory $D_t p_i = L_{q^i}$, and $D_t H = -L_t$. Therefore

$$\begin{aligned} D_t(p_i\xi^i - H\tau - B) &= L_{q^i}\xi^i + p_i D_t\xi^i + L_t\tau - HD_t\tau - D_t B \\ &= L_t\tau + L_{q^i}\xi^i + p_i(D_t\xi^i - \dot{q}^i D_t\tau) + LD_t\tau - D_t B = 0. \end{aligned}$$

□

Definition 19 (Noether cup symmetry). A one-parameter variational symmetry is a symmetry of a cup datum $\mathfrak{C}$ when its flow preserves the constraint region C, maps admissible bounded-curvature paths to admissible paths, and satisfies the hypothesis of Theorem 18. The corresponding J_X is the conserved cup current.

This gives the exact general form:

$$\boxed{\text{closed variational symmetry} \longrightarrow \text{conserved current through the cup.}}$$

The symmetry and the current are dual offices of one theorem. They are not two names for the same object.

VIII. Axial turn symmetry and orbit

Take $\mathcal{Q} = \mathbb{R}^2 \setminus \{0\}$, let $J_0 := -\Omega = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$ be the counterclockwise quarter-turn, and let

$$L(q,\dot{q}) = \frac{m}{2}\|\dot{q}\|^2 - U(\|q\|)$$

for a C^1 central potential U. The rotation group $R_\theta = e^{\theta J_0}$ preserves both L and every radial cup $C_R = \{q : 0 < \|q\| \le R\} \subset \mathcal{Q}$.

Corollary 20 (Axial turn symmetry). *For the infinitesimal generator $\xi(q) = J_0 q$, with $\tau = B = 0$, Theorem 18 gives the conserved angular current*

$$\boxed{\mathcal{L}_z = p^{\mathsf{T}} J_0 q = m(q_1\dot{q}_2 - q_2\dot{q}_1)}, \qquad \frac{\mathrm{d}\mathcal{L}_z}{\mathrm{d}t} = 0.$$

Proof. The radial potential and Euclidean kinetic energy are rotation-invariant, so the Noether hypothesis holds with $B = 0$. Substitute $\xi = J_0 q$. □

For a fixed-radius cup $q(t) = R(\cos\theta(t), \sin\theta(t))$, one obtains

$$\mathcal{L}_z = mR^2\dot{\theta}.$$

Thus an unbroken axial symmetry forces constant angular velocity on the fixed radius. This is the minimum rotational endurance statement: orientation turns while the axial current remains constant.

For the inverse-square potential $U(r) = -\mu m/r$, the circular solution

$$q(t) = R\big(\cos(\omega t), \sin(\omega t)\big), \qquad \omega^2 = \frac{\mu}{R^3},$$

has curvature $1/R$, period $2\pi/\omega$, and conserved $\mathcal{L}_z = mR^2\omega$. It is a closed rotational cup and a classical orbit. Return of position after one period does not say that no motion occurred; the path and phase supply the excursion.

IX. SPIN AND IDENTITY RETURN

Let

$$U_{1/2}(\theta) := \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix} \in SU(2).$$

Under the standard double cover $SU(2) \to SO(3)$, this is rotation by θ about a fixed axis [7].

Proposition 21 (Double return).

$$U_{1/2}(2\pi) = -I_2, \qquad U_{1/2}(4\pi) = I_2.$$

For the represented difference $\mathfrak{D}(U) = U - I_2$,

$$\mathfrak{D}_{2\pi} = -2I_2, \qquad \mathfrak{D}_{4\pi} = 0.$$

The second equality follows independently from the nonlinear composition law:

$$(-2I_2) + (-2I_2) + (-2I_2)^2 = 0.$$

The physical rotation has returned after 2π in $SO(3)$ while the spin lift has not. After 4π the represented endpoint also returns. A receipt carrying the parameterized path distinguishes that double turn from a constant path even though both reduced endpoint reads are I_2 and 0. This is identity through return, not identity by flattening the lift.

X. TIME ROTATION AND THE TWENTY-EIGHT DIFFERENCES

Let

$$P_4 := \{0,1,2,3\}, \qquad \mathrm{Phase}_4(n) := n \bmod 4, \qquad \mathrm{Cycle}_4(n) := \left\lfloor \frac{n}{4} \right\rfloor.$$

Define

$$\mathrm{TimeRead}_4(n) := (\mathrm{Phase}_4(n), \mathrm{Cycle}_4(n)), \qquad \mathrm{UnRead}_4(p,c) := 4c + p.$$

Euclidean division gives a bijection $\mathbb{N} \cong P_4 \times \mathbb{N}$ and an exact successor-with-carry law.

Fix a run identifier u and define

$$o_n := (u,n), \qquad \tau_n := (o_n, o_{n+1}), \qquad \mathcal{T}_n := (\tau_0, \dots, \tau_{n-1}).$$

Then

$$\mathrm{Phase}_4(n+4) = \mathrm{Phase}_4(n), \qquad \mathrm{Cycle}_4(n+4) = \mathrm{Cycle}_4(n) + 1, \qquad o_{n+4} \neq o_n, \qquad \mathcal{T}_n \prec \mathcal{T}_{n+4}.$$

The wheel phase returns; the occurrence does not.

Take two successive four-phase lifts. They supply eight occurrence addresses, indexed by $i = 4\ell + p$ with $\ell \in \{0,1\}$ and $p \in P_4$. Let $E_8 = \mathbb{R}^8$ with basis $e_0, \dots, e_7$, and let

$$\Omega_8 = \sum_{0 \le i < j \le 7} d_{ij} e^i \wedge e^j.$$

Since

$$\dim \Lambda^2(E_8^*) = \binom{8}{2} = 28,$$

there is one signed coordinate for every unordered pair of occurrences. Define

$$\zeta_{\Omega_8}(e_i, e_j) := \Omega_8(e_i, e_j).$$

Then

$$\zeta_{\Omega_8}(e_j, e_i) = -\zeta_{\Omega_8}(e_i, e_j), \qquad \zeta_{\Omega_8}(e_i, e_i) = 0.$$

The zero is a diagonal comparison value, not an empty type or absent read.

For an oriented triangle define

$$h_{ijk} := d_{ij} + d_{jk} + d_{ki}.$$

Proposition 22 (Triangle closure). *The following are equivalent:*

(i) $h_{ijk} = 0$ *for every oriented triangle;*

(ii) there exist vertex values s_i*, unique up to a common additive constant, such that* $d_{ij} = s_j - s_i$*.*

Proof. A vertex potential telescopes around every triangle. Conversely, fix $s_0 = 0$ and set $s_i = d_{0i}$; the triangle $(0, i, j)$ gives $d_{ij} = d_{0j} - d_{0i}$. □

To scan all seams once, label the eight vertices by $\{\infty\} \cup \mathbb{Z}_7$ and set

$$M_a := \{\{\infty, a\}\} \cup \{\{a + j, a - j\} : j = 1, 2, 3\}, \qquad a \in \mathbb{Z}_7.$$

The seven M_a are edge-disjoint perfect matchings and partition $E(K_8)$. With inner coordinate $b \in \mathbb{Z}_4$, the Chinese remainder theorem gives

$$\mathbb{Z}_{28} \cong \mathbb{Z}_7 \times \mathbb{Z}_4,$$

and an explicit bijective scanner

$$\mathrm{Scan}_{28} : \mathbb{Z}_{28} \xrightarrow{\sim} E(K_8).$$

The display angle may be declared by

$$\Theta_s = \Theta_0 + s\frac{\pi}{14}, \qquad \theta_s = \Theta_s \bmod 2\pi.$$

This is an angular read, not a duration law.

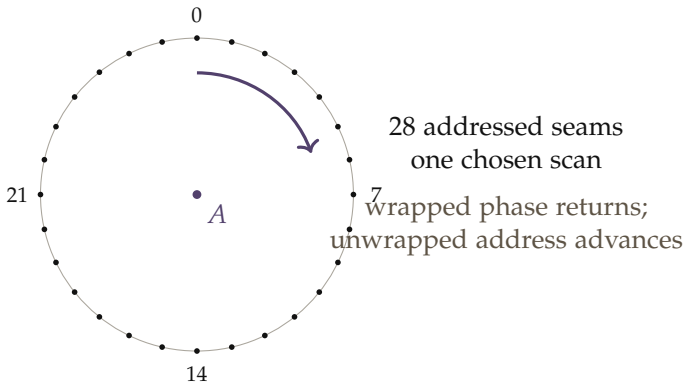


Figure 2: The twenty-eight-difference clock. The center is a read aperture, not a ninth occurrence and not the alternating diagonal zero.

More generally, for any abelian carrier W and addressed path $x_0, \ldots, x_{28}$, let $\delta_s = x_{s+1} - x_s$. If $x_{28} = x_0$, then

$$\sum_{s=0}^{27} \delta_s = x_{28} - x_0 = 0,$$

while the receipt sequence still contains twenty-eight occurrences. This is return without amnesia.

XI. Infinity return, horizon, and Tomorrow

The projective action of Q on the affine real line is

$$T(x) = 1 + \frac{1}{x}.$$

Its fixed points are Φ and Ψ. Starting at the projective point ∞,

$$T^n(\infty) = \frac{F_{n+1}}{F_n} \qquad (n \geq 1).$$

Theorem 23 (Positive unattained horizon). *The finite orbit satisfies*

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \Phi,$$

but

$$\frac{F_{n+1}}{F_n} \neq \Phi \qquad \text{for every finite } n \geq 1.$$

Thus Φ is a positive fixed horizon of the rational Fibonacci orbit: it orients the continuation and is approached by every sufficiently advanced read, but no finite rational address occupies it.

Proof. Binet's formula gives convergence. Every finite ratio is rational, whereas Φ is irrational. □

The inverse branch $\Psi = -\Phi^{-1}$ is carried simultaneously. Mirror $H_{\sqrt{5}}$ exchanges the two projective null rays without identifying them. The lift line and return line are therefore one mirror-held boundary pair of the golden cup.

Define the successor

$$S : \mathbb{N} \to \mathbb{N}, \qquad S(n) = n + 1.$$

It has no fixed point and $\mathbb{N}$ has no greatest element.

Definition 24 (Tomorrow). At present address n, define

$$\text{Tomorrow}(n) := S(n) = n + 1.$$

Then $\text{Tomorrow}(n) \neq n$. After the successor is realized, the new present is $n + 1$ and its Tomorrow is $n + 2$.

Tomorrow is therefore not an unknown event-content oracle. It is the exact positive horizon of local ordinal continuation: always one admissible address ahead, never a fixed point of the present-address operator.

XII. Explicit light-flow witness

In the standard differential-form description of Abelian gauge theory [8, 9, 10], let

$$M = \mathbb{R}_t \times S^1_\vartheta \times \mathbb{R}_x \times \mathbb{R}_y$$

and let η be the standard closed one-form on S^1, normalized by $\oint_{S^1} \eta = 2\pi$. For constants $c, \rho > 0$, define

$$g = -c^2\,\mathrm{d}t^2 + \rho^2\eta^2 + \mathrm{d}x^2 + \mathrm{d}y^2.$$

Let $E = M \times \mathbb{R}^2$ carry the $SO(2)$ generator Ω, and for $f \in C^2(\mathbb{R})$ with $f' \not\equiv 0$ define

$$u = t - \frac{x}{c}, \qquad A_a = a\eta + f(u)\,\mathrm{d}y, \qquad a \in \mathbb{R}.$$

Theorem 25 (Source-free null flow). *The curvature is*

$$F = \mathrm{d}A_a = f'(u)\,\mathrm{d}u \wedge \mathrm{d}y,$$

independent of a, nonzero for nonconstant f, and satisfies

$$\mathrm{d}F = 0, \qquad \mathrm{d}{*_g}F = 0.$$

The curves

$$\gamma_{x_0}(\lambda) = (\lambda, [0], x_0 + c\lambda, 0)$$

are future-directed null lines:

$$g(\dot\gamma_{x_0}, \dot\gamma_{x_0}) = 0, \qquad u(\gamma_{x_0}(\lambda)) = -x_0/c.$$

Thus the wave profile is transported unchanged along the null flow.

Proof. Since the gauge algebra is Abelian, $F = \mathrm{d}A_a = f'(u)\,\mathrm{d}u \wedge \mathrm{d}y$. Closure is immediate. In the constant metric, the only nonzero components are $F_{ty} = f'(u)$ and $F_{xy} = -f'(u)/c$; their divergence cancels:

$$\partial_t F^{ty} + \partial_x F^{xy} = -\frac{1}{c^2} f''(u) + \frac{1}{c^2} f''(u) = 0.$$

The null-line computation is $-c^2(1)^2 + (c)^2 = 0$, and u is constant along γ_{x_0}. □

For a loop ℓ winding once around S^1 at fixed t, x, y,

$$\mathrm{Hol}_{A_a}(\ell) = e^{-2\pi a \Omega}.$$

Hence local curvature depends only on f, while global holonomy depends only on a; the two are independently tunable. The same quarter-turn generator that realizes the finite wheel now occupies an $SO(2)$ parallel-transport office, while the scalar profile remains independently chosen.

The null lines are the physical line-symmetry witness of this paper. Under the linear normal form of Section III, the algebraic golden rays are null lines of a signature-$(1,1)$ carrier; under Theorem 25, a Maxwell profile propagates on null lines of a Lorentzian spacetime. These are joined by an explicit common type—null line under an indefinite metric—not by an equation $\Phi = c$ or by a claim that the Fibonacci matrix determines the physical speed of light.

For timelike states the null boundary is an unobtainable velocity horizon; for the Maxwell field it is the admitted carrier of propagation. This is the precise two-office form of the positive horizon: a limiting boundary for one class and a lawful flow line for another.

XIII. Proof of the main theorem

The tuple in Theorem 1 is now explicit. Items (i)–(iii) are Theorem 3 and Proposition 5. Item (iv) is Theorem 6. Item (v) is Theorems 8 and 9. Item (vi) is the Memory Return construction and Theorem 14. Item (vii) is the cup datum and its mirror specialization. Item (viii) is Theorem 18. Item (ix) is Corollary 20 and the explicit circular orbit. Item (x) is Proposition 21. Item (xi) is the four-phase clock, eight-address window, two-form reader, and one-factorization. Items (xii)–(xiii) are Theorem 23 and the successor construction. Item (xiv) is Theorem 25. Every listed component is therefore inhabited, and all non-identifications stated in the theorem are visible from their distinct domains and codomains. □

XIV. Strict boundary and reuse ledger

The following cuts are part of the result.

(a) ζ_ω and ζ_{Ω_8} are addressed difference readers, not the Riemann zeta function; neither has a scalar inverse on its image without extra address data.

(b) $H_{\sqrt{5}} = G/\sqrt{5}$ is the unique live name for the golden Euclidean mirror in this paper.

(c) $Q^2 = \mathcal{R}_Q$ restores the dual-form relation. It is not an append operation and is not a definition of memory in general.

(d) A receipt may retain an event without recurrence; a return reader may close while the receipt remains nontrivial. Bookkeeping and return are separately typed.

(e) 0, $\bot$, Unknown, and Unavailable are non-coercible: a zero read, an uninhabited type, insufficient evidence, and an unposed read are different judgments.

(f) Noether conservation is used only under the displayed variational hypothesis. Axial angular momentum is its proved specialization, not a free slogan.

(g) Equal phase or projective reads do not imply equality of the complete occurrence-bearing state.

(h) The ideal horizon Φ is a projective fixed point; no finite rational iterate is identified with it. Tomorrow is successor, not prediction.

(i) The golden null lines and Lorentzian light lines share an exact null-line normal form. The paper does not equate $\sqrt{5}$ with c or derive the Maxwell field from Fibonacci arithmetic.

(j) Curvature, holonomy, Noether charge, proper time, phase, receipt history, and projective address remain independently typed. Equality in one channel cannot be substituted for equality in another.

(k) Endpoint recurrence alone does not license Berry phase. A geometric phase would require a parameter path, a common Hilbert bundle, a regular gapped spectral family, a connection, and an endpoint or loop reader.

The reuse rule is consequently simple: a theorem may be carried forward only with its carrier, hypotheses, operator, invariant, and codomain intact. A shared name or matching value is not a transport witness.

Mechanical audit

The companion script `check_minimum_endurance_delta_v01.py` executes 164 exact symbolic and finite-combinatorial checks: the dual-form identities, reflection factorization, golden basis, Fibonacci powers, finite horizon non-attainment, the complete K_8 one-factorization, four-phase non-erasing return, spin double return, and axial-current conservation. Its terminal result is `PASS exact regression suite (164 checks)`. The script is a regression gate for the displayed identities, not a substitute for the universal proofs in the text.

Minimum compression

$$
\begin{gathered}
Q^*\omega = -\omega, \qquad Q^* g_Q = -g_Q, \qquad (Q^2)^*\omega = \omega, \qquad (Q^2)^* g_Q = g_Q, \\
\zeta_\omega(v_\Phi, v_\Psi) = \sqrt{5}, \qquad hqh = q^{-1}, \qquad \langle q, h\rangle \cong D_\infty, \\
\delta_X S = 0 \Longrightarrow D_t J_X = 0, \qquad \frac{\mathrm{d}\mathcal{L}_z}{\mathrm{d}t} = 0, \\
\mathrm{Phase}_4(n+4) = \mathrm{Phase}_4(n), \qquad o_{n+4} \neq o_n, \\
\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \Phi, \qquad S(n) = n+1 \neq n.
\end{gathered}
$$

Motion supplies the excursion. Mirror returns orientation.
Symmetry carries the current. Difference keeps the address open.

Definition register

Name	Carrier / normal form	Exact office
Fibonacci carriage	$Q = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right) : \mathbb{R}^2 \to \mathbb{R}^2$	$Q^2 = Q + I$; Q^n carries Fibonacci coefficients.
Orientation form	$\omega(u, v) = u^{\mathsf{T}} \Omega v$, $\Omega^2 = -I$	Alternating addressed difference; quarter-turn after Euclidean identification.
Symmetric difference	$g_Q(u, v) = \omega(Qu, v) - \omega(u, Qv)$	Derived form with matrix $G = [Q, \Omega]$.
Dual-form return	$\mathcal{R}_Q = Q^2$	$(Q^2)^* \omega = \omega$, $(Q^2)^* g_Q = g_Q$; state return not inferred.
Golden roots	$\Phi = (1 + \sqrt{5})/2$, $\Psi = (1 - \sqrt{5})/2$	Eigenvalues and projective null rays; $\zeta_\omega(v_\Phi, v_\Psi) = \sqrt{5}$.
Golden mirror	$H_{\sqrt{5}} = G/\sqrt{5}$	Euclidean reflection; exchanges golden rays; projectively conjugates q to q^{-1}.
Finite wheel	$\langle \Omega, H_{\sqrt{5}} \rangle \cong \mathrm{Dih}_4$	Four-phase orientation and mirror return.
Infinite motion	$\langle q, h \mid h^2 = 1, hqh = q^{-1} \rangle \cong D_\infty$	Integer-addressed carriage with inverse orientation.
ζ read	$\zeta_\omega : V \times V \to \mathbb{R}$; $\zeta_{\Omega_8} : E_8 \times E_8 \to \mathbb{R}$	Signed returned difference; no scalar inverse.
Memory Return	$(r, x, \gamma, \widetilde{\gamma}, \varepsilon, \eta, \rho, \mathfrak{D}_\rho(\varepsilon), \mathcal{R})$	Reader return + lift + represented difference + lossless receipt.
Cup	$(M, g_R, C, I, \gamma, \kappa_{\max}, r, \eta_{\text{return}})$	Bounded curvature held inside an admissible motion relation.
Noether current	$J_X = p_i \xi^i - H\tau - B$	Conserved when the displayed variational-symmetry identity holds.
Axial current	$\mathcal{L}_z = p^{\mathsf{T}} \Omega q$	Angular momentum conserved under radial cup symmetry.
Spin return	$U_{1/2}(2\pi) = -I$, $U_{1/2}(4\pi) = I$	Physical rotation may return before its spin lift; double return closes represented endpoint.
Occurrence	$o_n = (u, n)$	Stable event address; matching phase does not identify occurrences.
Time rotation	$\mathbb{N} \cong P_4 \times \mathbb{N}$	Four-phase read with cycle carry; phase return and ordinal advance coexist.
Twenty-eight delta	$\dim \Lambda^2((\mathbb{R}^8)^*) = 28$	All pairwise seams of two four-phase lifts; seven-by-four scan.
Identity	Equality in a declared carrier or reader	Reader-relative return is not automatically full-state identity.
Horizon	$\lim F_{n+1}/F_n = \Phi$, finite ratios $\neq \Phi$	Positive fixed orientation approached but not occupied by finite rational address.
Tomorrow	$S(n) = n + 1$	No fixed point and no terminal natural-number address; next ordinal, not event prediction.
Light flow	$F = f'(t - x/c)\ \mathrm{d}(t - x/c) \wedge \mathrm{d}y$	Nonzero source-free Maxwell curvature transported along future null lines.
Silence	Exact zero in a declared read while the event receipt remains present	Zero observable does not imply blank occurrence.
Endurance	Invariant relation carried through admitted motion and return	Symmetry closes; current, address, or receipt supplies continuation without erasure.

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Motion supplies the excursion. Difference makes return readable.
Symmetry carries the current. The next address remains open.