

MIRROR ESCAPEMENT TIME LIFT

$$D(F^4(r, z)) = D(r, z), \quad \Lambda(L_{\text{ax}}z) = \Lambda(z) + T$$

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Abstract. This paper consolidates an exact bridge cycle already distributed across the dihedral phase, minimum-root, axial-return, central-lift, and mechanical-receipt results. The root pair Q, C_1 generates the complete local algebra $M_2(\mathbb{R})$. Its normalized mirror produces a finite four-phase wheel and an infinite projective Fibonacci carriage, with the former an exact quotient of the latter. The positive axial carriage is then rectified to translation by a cover coordinate Λ ; Euler exponentiation turns that translation into a circular reader; and a four-state escapement converts four visible phase advances into one nontrivial deck step. For every $m \geq 0$, the resulting dynamics satisfies

$$F^{4m}(r, z) = (r, L_{\text{ax}}^m z), \quad D(F^{4m}(r, z)) = D(r, z), \quad \Lambda(L_{\text{ax}}^m z) = \Lambda(z) + mT.$$

The ring closes; the lifted address continues. A companion $SU(2)$ theorem shows independently that a nontrivial turning symmetry which inverts a circle has forced square $-I$, while its $SO(3)$ projection is involutive. The result is a typed algebraic dynamics of mirror reversal, counted release, visible return, and retained traversal.

Exact scope

No new empirical dynamics is asserted here. The paper composes previously proved mathematical results, derives their complete escapement iteration law, and fixes the type boundaries between phase, lifted address, receipt, and physical realization. Distinct carriers are connected only through the displayed maps.

I. Engine identity

The object is not the circular phase read alone. Phase is motion read modulo return. The engine is the operation which lets that compact read close while an ordered lift advances.

Its four offices are

$$\begin{aligned} \text{Mirror} &: HAH = A^{-1}, \\ \text{Escapement} &: F^4(r, z) = (r, L_{\text{ax}}z), \\ \text{Time} &: \Lambda(L_{\text{ax}}z) = \Lambda(z) + T, \\ \text{Lift} &: p(L_{\text{ax}}z) = p(z), \quad L_{\text{ax}}z \neq z. \end{aligned}$$

The first line supplies reversible opposition. The second meters it. The third gives ordered advance. The fourth preserves the difference removed by the quotient. Their composition is called the *Mirror Escapement Time Lift*.

Engine statement

For a declared phase reader D , cover coordinate Λ , period $T > 0$, and escapement F , the engine closes after four local phases exactly when

$$D(F^4x) = D(x), \quad \Lambda(F^4x) = \Lambda(x) + T.$$

The first equality is the ring return. The second is the time lift. Neither cancels the other.

II. The root engine block

Let

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \Omega := -C_1,$$

and define the order defect

$$G := [C_1, Q] = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}.$$

The certified local identities are

$$C_1^2 = -I, \quad C_1^4 = I, \quad Q^2 = Q + I, \quad G^2 = 5I.$$

Moreover,

$$\{I, Q, \Omega, G\}$$

is a basis of $M_2(\mathbb{R})$. Thus two generators produce the complete local real 2×2 matrix algebra, while every evaluated word has a unique four-coordinate present-state form.

Write

$$\Phi = \frac{1 + \sqrt{5}}{2}, \quad \Psi = \frac{1 - \sqrt{5}}{2} = -\Phi^{-1},$$

and normalize

$$H := \frac{G}{\sqrt{5}}, \quad S := HC_1 = \frac{2Q - I}{\sqrt{5}}.$$

Then

$$H^2 = S^2 = I, \quad HS = C_1, \quad SH = -C_1, \quad \{H, S\} = 0.$$

The matrix H is the root mirror; S is the normalized axial generator. The positive carriage is

$$L_{\text{ax}} := \frac{Q + 2I}{\sqrt{5}} = e^{(\log \Phi)S}, \quad L_{\text{ax}}^2 = Q^2.$$

This is the engine block: orientation, transport, generated difference, mirror, axial generator, and positive carriage all close inside one exact algebra.

III. Mirror gearbox: finite ring and infinite carriage

The same mirror has two separately typed dihedral actions.

First,

$$H^2 = I, \quad C_1^4 = I, \quad HC_1H = C_1^{-1},$$

so

$$\langle C_1, H \rangle \cong D_4,$$

where $|D_4| = 8$. This is the finite orientation wheel.

Second, direct multiplication gives

$$HQH = -Q^{-1}.$$

For projective classes $q = [Q]$ and $h = [H]$, the scalar sign disappears:

$$h^2 = 1, \quad hqh = q^{-1}.$$

Because the projective eigenvalue ratio of Q is $-\Phi^2$, the element q has infinite order. There-

fore

$$\langle q, h \rangle \cong D_\infty.$$

The finite ring is not merely similar to the infinite carriage. The generator assignment

$$\pi_4 : D_\infty \longrightarrow D_4, \quad \pi_4(q) = C_1, \quad \pi_4(h) = H,$$

is a surjective homomorphism with

$$\ker \pi_4 = \langle q^4 \rangle.$$

Hence

$$\boxed{D_\infty / \langle q^4 \rangle \cong D_4.}$$

The four-phase ring is the quotient. Infinite Fibonacci motion is the addressed lift. One complete visible revolution is exactly the point at which the quotient can no longer display which lift occurred.

IV. Axial rectification and the time coordinate

Let $\mathbb{H}^+$ be the upper half-plane. Define the golden projective chart

$$\xi_{\text{ax}}(z) = \frac{\Phi - z}{z - \Psi}$$

and the centered cover coordinate

$$\Lambda(z) = \log \left(\frac{\Phi}{|\xi_{\text{ax}}(z)|} \right), \quad T := 2 \log \Phi.$$

For the continuous carriage

$$L_t := e^{t(\log \Phi)S}, \quad t \in \mathbb{R},$$

one has

$$\xi_{\text{ax}}(L_t z) = \Phi^{-2t} \xi_{\text{ax}}(z), \quad \boxed{\Lambda(L_t z) = \Lambda(z) + tT.}$$

Let

$$p : \mathbb{H}^+ \longrightarrow M_{\text{ax}} := \langle L_{\text{ax}} \rangle \backslash \mathbb{H}^+$$

be the quotient. Translation by T closes under p , while it remains nonzero on the cover. The differential $d\Lambda$ descends to a closed one-form $\bar{\alpha}_{\text{ret}}$ with

$$p^* \bar{\alpha}_{\text{ret}} = d\Lambda, \quad \oint_{\text{core}} \bar{\alpha}_{\text{ret}} = T.$$

The quotient closes; the harmonic period retains the traversal.

Normalize the period and exponentiate:

$$W(p(z)) := \exp\left(\frac{2\pi i \Lambda(z)}{T}\right).$$

Then

$$W(p(L_t z)) = e^{2\pi i t} W(p(z)).$$

Thus the unbounded axial coordinate becomes a circle-valued reader without being replaced by it. In particular,

$$W(p(L_{1/2} z)) = -W(p(z)), \quad W(p(L_1 z)) = W(p(z)).$$

Euler supplies the dial. The cover coordinate supplies the ordered lift.

V. Escapement dynamics

Let $P_4 = \mathbb{Z}/4\mathbb{Z}$ and

$$X_4 := P_4 \times \mathbb{H}^+.$$

Define the four-phase escapement

$$F(r, z) = \begin{cases} (r+1, z), & r = 0, 1, 2, \\ (0, L_{\text{ax}} z), & r = 3, \end{cases}$$

and the display

$$D(r, z) := (i^r, p(z)) \in \{1, i, -1, -i\} \times M_{\text{ax}}.$$

For $n \geq 0$, let

$$c_n(r) := \left\lfloor \frac{r+n}{4} \right\rfloor, \quad r \in \{0, 1, 2, 3\}.$$

The Mirror Escapement Time Lift theorem

For every $(r, z) \in X_4$ and every $n \geq 0$,

$$F^n(r, z) = ([r+n]_4, L_{\text{ax}}^{c_n(r)} z).$$

Consequently, for every $m \geq 0$,

$$\begin{aligned} F^{4m}(r, z) &= (r, L_{\text{ax}}^m z), \\ D(F^{4m}(r, z)) &= D(r, z), \\ \Lambda(L_{\text{ax}}^m z) &= \Lambda(z) + mT. \end{aligned}$$

Proof. Every successor step advances the residue r by one. The axial carriage is applied exactly when the residue crosses from 3 to 0. Among the first n steps, the number of such

crossings is $c_n(r)$. This gives the first formula. For $n = 4m$, the residue returns to r and the carry count is m . Since $p \circ L_{\text{ax}} = p$, the display returns. Iterating $\Lambda(L_{\text{ax}}z) = \Lambda(z) + T$ gives the final line. ■

The theorem is pure dynamics in algebraic form. The local state is finite. The carry is unbounded. The ring returns because the lift advances through the quotient seam.

VI. The central mirror lift

The root engine does not require $SU(2)$. The standard rotation cover nevertheless supplies an independent exact inhabitant of the same mirror/lift distinction.

Let $X \in \mathfrak{su}(2)$ satisfy $X^2 = -I$, and put

$$U(t) = e^{tX} = \cos t I + \sin t X, \quad \mathcal{T} := U(\mathbb{R}) \cong U(1).$$

Suppose $g \in SU(2)$ is a nontrivial turning element satisfying

$$gU(t)g^{-1} = U(-t) \quad \text{for every } t.$$

Applying the relation twice shows that g^2 centralizes $\mathcal{T}$. The circle is self-centralizing in $SU(2)$, so $g^2 \in \mathcal{T}$. Conjugation by g inverts $\mathcal{T}$, while g commutes with its own square; hence

$$g^2 = (g^2)^{-1}, \quad g^2 \in \{\pm I\}.$$

The value $+I$ would make g a central involution, incapable of inverting a nontrivial circle. Therefore

$$\boxed{g^2 = -I = U(\pi), \quad g^4 = I.}$$

For the double cover $\varpi : SU(2) \rightarrow SO(3)$,

$$\varpi(g)^2 = \varpi(-I) = I.$$

The projection reads an involution; the lift retains the central half-turn. This is a central parity receipt, not a complete traversal count. After the next square, even the lifted endpoint closes. Lossless occurrence still requires a separate address or ledger.

VII. The gravity ring cycle

The exact engine read is the paired map

$$\mathcal{G}_{\text{METL}} : X_4 \longrightarrow (\{1, i, -1, -i\} \times M_{\text{ax}}) \times \mathbb{R}, \quad \mathcal{G}_{\text{METL}}(r, z) := (D(r, z), \Lambda(z)).$$

Under one completed escapement cycle,

$$\boxed{\mathcal{G}_{\text{METL}}(F^4(r, z)) = (D(r, z), \Lambda(z) + T).}$$

The first component is the ring read. The second is the axial lift. The escapement is the exact fit that lets one close while the other advances.

In the project vocabulary, *Gravity* names this Fit office: the coherent coupling of a compact public read to its noncompact carried address. *Light* names the circular display; *Motion* names the action F ; *Memory* names the retained period, address, and receipt. These office names do not identify the root algebra with Newtonian force, Einstein curvature, or any unsupplied physical realization.

The complete gravity ring cycle is therefore

$$\boxed{\text{Mirror} \longrightarrow \text{Escapement} \longrightarrow \text{Ring Return} \mid \text{Time Lift.}}$$

The vertical bar is the quotient seam. It separates the value that returns from the traversal that continues while retaining their exact coupling.

VIII. Memory, silence, and receipt

Let $\mathcal{R}_j$ be an append-only ledger with

$$\mathcal{R}_{j+1} = \mathcal{R}_j \frown o_{j+1}.$$

For $m > 0$, the completed engine read is the product-typed triple

$$(D(F^{4m}x) - D(x), \Lambda(F^{4m}x) - \Lambda(x), \mathcal{R}_{j+m}) = (0, mT, \mathcal{R}_{j+m}),$$

with

$$\mathcal{R}_{j+m} \neq \mathcal{R}_j.$$

The zero occupies the display-difference office. The nonzero period occupies the lifted-time office. The ledger occupies the occurrence office. They are read together and never added across types.

This is the engine form of Silence: exact readable balance without erased motion. The display has no residual phase difference; the carriage has advanced; the receipt can certify how many returns occurred.

IX. Status and boundary

Result	Status	Dependency or boundary
Complete local root algebra	Certified	Direct matrix reduction.
Finite D_4 mirror wheel	Certified	$C_1^4 = H^2 = I$, $HC_1H = C_1^{-1}$.
Infinite D_∞ Fibonacci carriage	Certified	Projective q has infinite order and $hgh = q^{-1}$.
$D_\infty / \langle q^4 \rangle \cong D_4$	Certified	Generator map and kernel calculation.
Axial translation coordinate	Certified	Golden rectification of L_t .
Euler circle reader	Derived	Integral period of $\bar{\alpha}_{\text{ret}}/T$.
Four-phase escapement carry	Certified	Declared skew-product update.
General n -step engine law	Derived here	Euclidean carry count $c_n(r)$.
Central $SU(2)$ square	Certified	Turning symmetry and centralizer lock.
Gravity as Fit office	Declared	Project vocabulary for the paired read; no physical-gravity identity.
Physical watch realization	Supplied by instance	Requires an actual oscillator, escapement, ratios, calibration, and receipt.

Non-coercion wall

The finite wheel, projective carriage, axial quotient, $SU(2)$ normalizer, and mechanical watch do not become one carrier because their return diagrams agree. The displayed maps are the adapters. A repeated word or glyph supplies no omitted map.

The parameter t , the cover coordinate Λ , the carry count m , and physical time measured by an apparatus remain distinct. The present theorem constructs the mathematical-mechanical bridge cycle. A particular physical clock enters only after its native mechanism and measurement protocol are supplied.

X. Final form

The full engine compresses to four lines:

$$\begin{aligned}
 HC_1H &= C_1^{-1}, & D_\infty / \langle q^4 \rangle &\cong D_4, \\
 \Lambda(L_t z) &= \Lambda(z) + tT, & W(p(L_t z)) &= e^{2\pi i t} W(p(z)), \\
 F^{4m}(r, z) &= (r, L_{\text{ax}}^m z), \\
 D(F^{4m}(r, z)) &= D(r, z), & \Lambda(L_{\text{ax}}^m z) &= \Lambda(z) + mT.
 \end{aligned}$$

Mirror reverses. Escapement meters. Time lifts.

The ring returns. The carriage continues. The receipt keeps the completed turn.

Provenance

The abstract dihedral constraint and the independent $SU(2)$ central-square proof were composed on 28 February 2026. The finite/infinite mirror quotient and central Silence architecture were consolidated on 30 July 2026. The complete local root algebra was presented on 10 August 2026; the axial-return theorem on 13 August 2026; and the Euler escapement construction on 17 August 2026. The present paper, dated 22 August 2026, introduces no replacement foundation. It gives those results their minimum dependency-ordered engine form and derives the displayed general iteration law.

References

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Mirror Escapement Time Lift

The ring returns. The address continues.