Physical Reality Inside Constraint

Gravity Contacting Light: an audited formal structure

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Abstract

This paper gives a receipt-preserving formal calculus with a typed address space and an append-only registry. The evaluation map $\mathrm{Ev} : P^* \to (\mathbb{N}_{>0}, \times)$ is the surjective, non-injective abelianization of ordered prime words; the root carrier

$$G = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$$

satisfies $G^2 = 5I$ and has split signature $(1,1)$; the inertia of $Q \otimes Q$ is $(2,2)$, while $G \oplus I_2$ has inertia $(3,1)$; the pp-wave curvature law, Fibonacci correction, and reduced Burau identities hold under their stated conventions and hypotheses; and the future-boundary registry $\widehat{K}_n = H_n \sqcup \{\mathbf{F}\}$ preserves historical receipts by append-only admission.

Its physical contact relation is

$$G_{\text{embodied}} = e, \qquad \Pi(e,\ell) = c, \qquad c = (e,\ell,w,r), \qquad \mathsf{Id}(c) = (e,\ell,w,r).$$

Here the embodied metric carrier e contacts light ℓ at an ordered, receipt-bearing address. The formal results, supplied structures, contact interfaces, and finite implementation are typed separately throughout.

Verification receipt. The companion program `verify_physical_reality_final.py` uses only Python's standard library. Every pass/fail assertion uses exact arithmetic in $\mathbb{Q}(\sqrt{5})$, Laurent polynomials, or a finite jet algebra. It verifies the displayed algebraic identities deterministically; its captured output is part of this edition's audit record.

1 Scope, status, and notation

1.1 Assertion discipline

Five different kinds of assertion occur below.

STATUS	MEANING
EXACT	A definition, calculation, or proof stated in this note.
POSTULATE	Supplied structure or a chosen specialization carried as an input.
CONTACT	A typed relation linking formal symbols to their physical reading.
IMPLEMENTATION	A finite realization that witnesses the stated relation.
PENDING AT **F**	A candidate held at the boundary object **F**, awaiting admission into the historical registry.

This separation keeps proof, supplied input, physical reading, computation, and pending occurrence in their own offices.

1.2 Root data

Let

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad G = [Q,\Omega] = Q\Omega - \Omega Q = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}. \tag{1}$$

Write

$$\Phi = \frac{1+\sqrt{5}}{2}, \qquad \Psi = \frac{1-\sqrt{5}}{2} = -\Phi^{-1}, \qquad F_1 = F_2 = 1. \tag{2}$$

Here $[A, B]$ denotes the matrix commutator.

2 Contact, addresses, and the future boundary

2.1 Prime-word address space

Let P be the set of prime numbers and let $W = P^*$ be the free monoid of finite ordered words in P, with concatenation as product and empty word ε as unit. A word $w = (p_1, \ldots, p_m)$ is an address history, not merely the integer it evaluates to.

Definition 2.1 (Evaluation and compression)**.** *Define*

$$\mathrm{Ev} : W \longrightarrow (\mathbb{N}_{>0}, \times), \qquad \mathrm{Ev}(p_1, \ldots, p_m) = \prod_{j=1}^{m} p_j, \quad \mathrm{Ev}(\varepsilon) = 1. \tag{3}$$

Theorem 2.2 (Exact compression theorem)**.** *The map* Ev *is the abelianization of the free monoid W: it is a surjective monoid homomorphism onto the free commutative monoid on P and is not injective.*

Proof. Concatenation becomes multiplication under (3), so Ev is a monoid homomorphism. Unique prime factorization gives a prime word for every positive integer, so it is surjective [1]. It identifies every pair of words that differ by adjacent exchanges, hence factors through the universal commutative quotient of P^*. Conversely, equality in the target fixes the multiplicity of each prime and therefore is exactly the equivalence generated by those exchanges. Finally,

$$\mathrm{Ev}(2,3) = 6 = \mathrm{Ev}(3,2), \qquad (2,3) \neq (3,2). \tag{4}$$

Thus the map is not injective. □

Equation (4) gives the formal compression read: order belongs to the source W, while multiplication belongs to its commutative numerical image. EXACT

2.2 Contact as a typed record

The physical reading used by this project is retained as a declared interface:

$$G_{\text{embodied}} = e, \qquad \Pi(e, \ell) = c, \qquad c = (e, \ell, w, r). \tag{5}$$

Here e names the embodied metric carrier, ℓ names light, $w \in W$ is the ordered prime-word address, and r is an append-only occurrence receipt. For this formal system the identity record is

$$\mathsf{Id}(c) = (e, \ell, w, r). \tag{6}$$

The numerical observation $\mathrm{Ev}(w)$ is one forgetting map out of that record; w and r remain carried by c. Equation (5) fixes the physical contact relation through which the formal system reads embodied gravity contacting light. CONTACT

2.3 Closed boundary and admission

Let $K_n = (c_1, \ldots, c_n)$ be an append-only finite sequence of contact records and let $H_n = \{c_1, \ldots, c_n\}$ be its historical carrier. The formal boundary completion is

$$\widehat{K}_n = H_n \sqcup \{\mathbf{F}\}, \tag{7}$$

where $\sqcup$ is disjoint union and $\mathbf{F} \notin H_n$ is the future-boundary object. A candidate is typed at $\mathbf{F}$ until an externally supplied predicate grants admission. More explicitly, for a candidate datum d let

$$\mathsf{Admit}(K_n, d) \in \{\mathsf{can}, \mathsf{refuse}\}. \tag{8}$$

If the value is can, a fresh receipt r_{n+1} is issued, $c_{n+1} = (e, \ell, w, r_{n+1})$, and $K_{n+1} = K_n \parallel c_{n+1}$; otherwise $K_{n+1} = K_n$ and the datum remains at $\mathbf{F}$.

Proposition 2.3 (Boundary invariant)**.** *Under the definition above, every issued contact record is historical (an element of H_n), while every unadmitted candidate remains at $\mathbf{F}$; no admitted receipt rewrites an earlier member of K_n.*

Proof. This is immediate by induction on the update rule following (8). The only successful update appends one new record; the refusal branch leaves the registry unchanged. □

The boundary form (7) and the predicate in (8) are supplied structure. They give the registry a fixed type: sequence for history, set carrier for its members, and a distinct boundary object for pending candidates. POSTULATE

2.4 Finite four-face admission policy

For the deterministic implementation receipt, define the ordered face set

$$\mathcal{F}_4 = (\mathsf{Color}, \mathsf{Meaning}, \mathsf{Memory}, \mathsf{Motion})$$

and its cyclic successor by

$$\mathsf{Color} \mapsto \mathsf{Meaning} \mapsto \mathsf{Memory} \mapsto \mathsf{Motion} \mapsto \mathsf{Color}.$$

Write f_n for the nth member of this cycle, with $f_0 = \mathsf{Color}$. The finite policy used in V10 is

$$\mathsf{Admit}_4(K_n, (f, e, \ell, w)) = \mathsf{can} \quad \Longleftrightarrow \quad f = f_n.$$

On admission it issues the receipt $r_{n+1} = (f, n)$ and appends $c_{n+1} = (e, \ell, w, r_{n+1})$ to K_n. All other candidates remain at $\mathbf{F}$. This is one supplied policy realizing the generic admission form (8). POSTULATE + IMPLEMENTATION

3 Mirror carrier and exact discrete witnesses

Theorem 3.1 (Root carrier)**.** *The carrier G in (1) satisfies*

$$G^2 = 5I, \qquad \det G = -5, \tag{9}$$

and has eigenvalues $\pm\sqrt{5}$ and signature $(1, 1)$.

Proof. Direct multiplication gives

$$\begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}^2 = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}.$$

The characteristic polynomial is $\lambda^2 - 5$, giving one positive and one negative eigenvalue. □

Proposition 3.2 (Null-coordinate normal form)**.** *Set*

$$n_+ = \begin{pmatrix} 1 \\ -\Psi \end{pmatrix}, \qquad n_- = \begin{pmatrix} 1 \\ -\Phi \end{pmatrix}, \qquad M = \left[n_+ \,\middle|\, -\frac{1}{5} n_- \right]. \tag{10}$$

Then

$$n_+^{\mathsf{T}} G n_+ = n_-^{\mathsf{T}} G n_- = 0, \qquad n_+^{\mathsf{T}} G n_- = -5, \tag{11}$$

and

$$M^{\mathsf{T}} G M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \det M = \frac{1}{\sqrt{5}}. \tag{12}$$

Thus the quadratic form of G has the null-coordinate read $2\,du\,dv$.

Proof. Substitute $\Phi + \Psi = 1$ and $\Phi\Psi = -1$ into the three quadratic expressions in (11); (12) then follows from bilinearity. EXACT □

Proposition 3.3 (Fibonacci correction)**.** *For every $n \geq 1$,*

$$F_{n+1} - \Phi F_n = \Psi^n, \qquad \left| \Phi - \frac{F_{n+1}}{F_n} \right| = \frac{|\Psi|^n}{F_n} = \frac{1 - (-1)^n \Phi^{-2n}}{\sqrt{5}\, F_n^2}. \tag{13}$$

Proof. The first identity follows by induction from the Fibonacci recurrence, or from Binet's formula. Divide it by F_n and use $F_n = (\Phi^n - \Psi^n)/\sqrt{5}$ to obtain the two exact error forms. □

The final factor in (13) approaches 1 but is not identically 1. This corrects the common but false replacement of the last expression by $1/(\sqrt{5}F_n^2)$. EXACT

Proposition 3.4 (A prime logarithmic near return)**.** *The ratio*

$$\frac{3^{12}}{2^{19}} = \frac{531441}{524288} \tag{14}$$

is not 1. More generally $\log 3/\log 2$ is irrational, so no nonzero integer powers of 2 and 3 coincide.

Proof. If $\log 3/\log 2 = a/b$ with nonzero integers a, b, then $3^b = 2^a$ after clearing signs, contrary to unique prime factorization. The stated ratio is an exact unequal rational number. EXACT □

4 Lorentzian interface and pp-wave boundary

Theorem 4.1 (Signature obstruction)**.** *The characteristic polynomial of $Q \otimes Q$ is*

$$\chi_{Q\otimes Q}(\lambda) = (\lambda + 1)^2(\lambda - \Phi^2)(\lambda - \Psi^2). \tag{15}$$

Hence $Q \otimes Q$ has inertia $(2, 2)$, while

$$G \oplus I_2 \quad \textit{has inertia} \quad (3, 1). \tag{16}$$

No real invertible congruence changes one of these forms into the other.

Proof. The real symmetric matrix Q has an orthonormal eigenbasis by the spectral theorem. Tensor products of those eigenvectors diagonalize $Q \otimes Q$; its eigenvalues are therefore $\Phi^2, \Phi\Psi, \Psi\Phi, \Psi^2$, namely two positive values and two copies of -1. The eigenvalues of $G \oplus I_2$ are $\sqrt{5}, -\sqrt{5}, 1, 1$. Sylvester's law of inertia preserves these positive and negative counts under real congruence [2]. □

Set $\widetilde{M} = M \oplus I_2$. Equation (12) gives the exact bridge

$$\widetilde{M}^{\mathsf{T}} (G \oplus I_2) \widetilde{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \oplus I_2.$$

In the coordinate order (u, v, x, y), this is the $H = 0$ Gram matrix of (17). Accordingly, $G \oplus I_2$ carries the flat signature $(3, 1)$ read of (12), while the inertia of $Q \otimes Q$ remains $(2, 2)$. The transverse block I_2 is the supplied extension carrying the planar mirror form into the Lorentzian carrier. EXACT + POSTULATE

For this section, coordinates are ordered (u, v, x, y) and signature is counted as $(\#+, \#-)$. Curvature uses

$$R_{bc} = \partial_a \Gamma^a_{bc} - \partial_c \Gamma^a_{ba} + \Gamma^a_{ad}\Gamma^d_{bc} - \Gamma^a_{cd}\Gamma^d_{ba},$$

and the field-equation convention is $G_{bc} = \kappa T_{bc}$. The symbol H is reserved in this note for the pp-wave profile. These are the Brinkmann pp-wave and Einstein-equation conventions used below [5, 6].

Proposition 4.2 (Conditional pp-wave identity)**.** *For the metric*

$$ds^2 = 2\,du\,dv + H(u,x,y)\,du^2 + dx^2 + dy^2, \tag{17}$$

one has $\det g = -1$*, scalar curvature* $R = 0$*, and*

$$R_{uu} = -\frac{1}{2}(\partial_x^2 + \partial_y^2)H, \tag{18}$$

with all other Ricci components zero. Thus, if the Einstein equation with this source ansatz is imposed, its uu *component reads*

$$\nabla_\perp^2 H = -2\kappa T_{uu}. \tag{19}$$

Proof. Invert the uv block of (17): $g^{uv} = 1$ and $g^{vv} = -H$. The only derivatives of the metric are derivatives of $g_{uu} = H$. Substitution in the Christoffel formula gives (18), and contraction gives $R = 0$. □

Equation (18) is the conditional differential-geometric identity carried by the pp-wave instance. Its profile H and source T_{uu} are specified pp-wave data. POSTULATE BOUNDARY

5 Braid–Burau read and the exact $\mathbb{Z}[\Phi]$ specialization

Use Artin's presentation

$$B_3 = \langle \sigma_1, \sigma_2 \mid \sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2 \rangle.$$

Both sides of the defining relation have exponent sum 3; hence $\sigma_i \mapsto 1$ descends to the exponent-sum homomorphism $\alpha : B_3 \to \mathbb{Z}$. Define the reduced Burau matrices over $\mathbb{Z}[t, t^{-1}]$ by [3, 4]

$$\psi(\sigma_1) = \begin{pmatrix} -t & 1 \\ 0 & 1 \end{pmatrix}, \qquad \psi(\sigma_2) = \begin{pmatrix} 1 & 0 \\ t & -t \end{pmatrix}. \tag{20}$$

Theorem 5.1 (Braid relation and determinant)**.** *The matrices in (20) satisfy*

$$\psi(\sigma_1\sigma_2\sigma_1) = \psi(\sigma_2\sigma_1\sigma_2) = \begin{pmatrix} 0 & -t \\ -t^2 & 0 \end{pmatrix}, \qquad \det\psi(w) = (-t)^{\alpha(w)}. \tag{21}$$

Proof. Multiply the two sides of the braid relation directly. Each generator has determinant $-t$, and the inverse generators have determinant $(-t)^{-1}$; the determinant formula follows multiplicatively. EXACT □

The specialization

$$t = \Phi^2 = 1 + \Phi \tag{22}$$

is the chosen parameter value carried by the specialization. POSTULATE

Proposition 5.2 (Exact specialization ring)**.** *At (22) the entries of* $\psi(\sigma_i^{\pm 1})$ *belong to*

$$\mathbb{Z}[\Phi] = \{a + b\Phi : a, b \in \mathbb{Z}\}, \qquad \Phi^2 = 1 + \Phi. \tag{23}$$

In particular,

$$\Phi^4 = 2 + 3\Phi, \qquad \Phi^{-1} = \Phi - 1, \qquad \Phi^{-2} = 2 - \Phi. \tag{24}$$

The number $\Phi^2 = (3 + \sqrt{5})/2$ *has half-integral* $\{1, \sqrt{5}\}$*-coordinates and integral* $\{1, \Phi\}$*-coordinates in* $\mathbb{Z}[\Phi]$*.*

Proof. The relation $\Phi^2 = 1 + \Phi$ gives the first part of (24); multiplying by the displayed inverses gives 1. In the basis $\{1, \sqrt{5}\}$, the coordinates of Φ^2 are half-integral, while $\{1, \Phi\}$ gives the integral coordinates displayed in (23). □

Proposition 5.3 (Orientation parity)**.** *At the specialization (22),*

$$\psi(\sigma_1)^2 = \begin{pmatrix} t^2 & 1-t \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \Phi^4 & -\Phi \\ 0 & 1 \end{pmatrix}. \tag{25}$$

It has eigenvalues Φ^4 *and* 1*. For every* $w \in B_3$*,*

$$\operatorname{sgn}\det\psi(w) = (-1)^{\alpha(w)}. \tag{26}$$

Proof. Square the first matrix in (20), use $1 - \Phi^2 = -\Phi$, and apply (21). Because $\Phi^2 > 0$, only the parity of the exponent sum contributes to the sign in (26). □

Equation (26) is the orientation-parity statement of this real matrix representation. The Möbius half-turn and physical-phase readings enter through their separately typed map. POSTULATE BOUNDARY

Proposition 5.4 (Why a receipt is needed)**.** *Let the generators act on a length-three prime word by adjacent swaps. The permutation read is its* S_3 *quotient, while* α *retains braid winding:* σ_1^2 *fixes* $(2, 3, 5)$ *while* $\alpha(\sigma_1^2) = 2$*.*

Proof. Two adjacent swaps cancel as permutations, whereas exponent sum is additive. The receipt carries the braid winding that lies beyond the permutation read. EXACT □

6 Address categories and loop interfaces

The following modest categorical conclusion is enough for the present note. If finite words are joined only by prefix inclusions, then the resulting category $\mathbf{Pats}_{\mathrm{pre}}$ is a poset: there is at most one morphism between any two objects. The position-covering sheaf read carries supplied covers and restriction maps. With strictly increasing subsequence embeddings, identical letters may carry more than one embedding. EXACT

The position-covering Grothendieck topology and all registry restriction maps are the supplied address interface. POSTULATE

For a standard gauge-theory loop interface, set $\hbar = 1$, let $q \in \mathbb{R}$ denote charge, and let $\phi = \rho e^{i\theta}$ on a closed loop γ, with $\rho \neq 0$ on that loop. Here $e^{i\theta}$ uses Euler's constant; the embodied carrier remains the distinct symbol e of (5). Let A be a connection one-form on a domain containing a spanning surface Σ with $\partial\Sigma = \gamma$, and write

$$F_A = dA, \qquad \Phi_B(\gamma;\Sigma) = \int_\Sigma F_A = \oint_\gamma A.$$

The final equality is Stokes' theorem on that domain. Then, for $n \in \mathbb{Z}$,

$$\oint_\gamma d\theta = 2\pi n, \qquad \oint_\gamma (d\theta - qA) = 2\pi n - q\oint_\gamma A = 2\pi n - q\Phi_B(\gamma;\Sigma). \tag{27}$$

It equals $2\pi k$ for $k \in \mathbb{Z}$ exactly when

$$q\Phi_B(\gamma;\Sigma) = 2\pi(n-k). \tag{28}$$

The nonvanishing hypothesis gives the loop a phase read. The golden ratio Φ of (2) and the flux Φ_B are distinct named quantities. The braid-word or receipt to physical-loop map belongs to the separately typed phase interface [7, 8]. EXACT + POSTULATE BOUNDARY

7 Deterministic implementation receipt

The accompanying verifier checks the following finite, reproducible results.

TEST	EXACT CHECK
V1–V2	The commutator G, $G^2 = 5I$, determinant, null vectors, and congruence (12).
V3–V4	Characteristic polynomial and inertia in (15)–(16), plus the diagonal transition calculation.
V5	The pp-wave inverse, determinant, Ricci tensor, and scalar curvature in (17)–(18).
V6	The braid relation and determinant formula over a fixed twelve-word suite.
V7–V8	Fibonacci identities, the exact comma ratio, and the word-action/winding separation.
V9	The $\mathbb{Z}[\Phi]$ identities, (25), and the determinant-sign parity statement.
V10	The supplied four-face policy Admit_4 admits only the next cyclic face and appends its receipt in that order.

V10 realizes the finite admission policy defined above. A passing test is the deterministic receipt that the stated finite formulas and receipt discipline were implemented correctly. IMPLEMENTATION

8 Closure ledger

STATEMENT	STATUS	REASON
Ordered prime words and Ev	exact	Theorem 2.2.
Contact tuple $c = (e, \ell, w, r)$	contact	It carries the embodied metric carrier, light, address, and receipt.
Future object **F** and admission	postulate	The supplied policy issues an append-only historical receipt.
$G^2 = 5I$ and null form	exact	Theorem 3.1 and Proposition 3.2.
Lorentzian flat carrier $G \oplus I_2$	exact plus postulate	Its signature is exact; the supplied I_2 completes the carrier.
pp-wave curvature equation	exact conditional identity	The metric (17) and Einstein equation carry its source law.
Burau orientation parity	exact after (22)	The chosen specialization carries the exact parity read.
Möbius/phase correspondence	postulate	Its typed map carries path data into braid and gauge data.
Detector, CIE, or real-field completion	pending at **F**	This address awaits its corresponding map and receipt.

9 Conclusion

The completed formal result is a receipt-preserving refinement placed before commutative evaluation. The exact map Ev gives the numerical read, while w and r preserve its ordered address and occurrence history. The matrix carrier supplies the split-signature algebraic normal form; the inertia calculation distinguishes the tensor carrier from the Lorentzian extension; and the Burau computation supplies an exact orientation-parity invariant at the chosen specialization.

The specification boundary is closed: historical records are in H_n, candidates are typed at **F**, and each future address carries its required map, hypothesis, and receipt condition. The contact relation $\Pi(e, \ell) = c$ remains present throughout this closure as the address of embodied gravity contacting light.

Source coordinates and audit provenance

The cited sources provide the standard constructions named in this note: prime factorization, matrix spectral and inertia results, braid presentation and Burau representation, pp-wave geometry, and gauge-loop phase. The audit record consists of this TeX source, the companion verifier `verify_physical_reality_final.py`, its captured output `verify_physical_reality_final_out.txt`, and the checksum record `physical_reality_release.sha256`. Together they provide the exact definitions, finite checks, and release receipt for this edition.

References

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