Symmetric Minimum

An Inherited Closure

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Abstract

This paper fixes one small exact system and says, for every statement in it, how much of that statement has been established. The core is a real 2×2 chart: a recurrence matrix Q, a quarter-turn J, a mirror H, and a golden carriage L_{ax}. These are exact and finite, and two independent computations agree on them. They show that returning to the same read and advancing to a new address are different operations. Around the core sits a vocabulary of twenty-eight words for how a relation is contacted, read, carried and recorded; each word has a typed map, a condition, a source coordinate and a status. The register closes on its last word, Endurance: after a half-turn the form survives, the difference remains, the reference returns and the read at zero balances. The field-facing layer joins the core, by a declared interface only, to the Einstein–Maxwell Hamiltonian closure of Minimum Mechanics 010k–010u. That closure is compiled in Lean 4.33.0 at pinned commits and has two tiers: exact pointwise results, and a conditional theorem that yields six brackets from seven named premises. The seven premises have no instance, and the Lean scalar type cannot carry one. No spectral statement is derived. Every statement carries the strength of its weakest named premise.

Page zero — what this is, in ordinary terms.

A *reader* is whatever takes up a relation and keeps a read of it: a person with a tally, an instrument with a calibration, a program with a log. Everything below is stated from the reader's side.

1. **Contact.** A relation meets a reader at one occurrence.
2. **Read.** The reader keeps a read at a stated resolution. Two inputs can give the same read while differing in what the reader needed; that failure is called *forget*.
3. **Receipt and memory.** The reader writes one entry that lets another reader rebuild that occurrence (the receipt). Separately, memory is how different occurrences stay comparable.
4. **Return and advance.** A read can return while the address moves on. The exact core makes this concrete: a four-step wheel returns its phase after one rotation, and in the same rotation the carried address takes one golden step.
5. **Strength.** A claim is as strong as its weakest named premise. The status tags (§1.1) say what kind of evidence stands behind each claim.

Where to start. The mathematics: §4–§5. The vocabulary: §3 and Appendix C. What is proved in Lean and what is not: §9 and Appendix E.

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1 Conventions

1.1 Status vocabulary

Every theorem, display and table row in this paper carries one of seven statuses. They are fixed here once and used without restatement.

Status	Meaning	Evidence it carries
Exact finite	Algebraic identity in a declared finite carrier, checked in exact symbolic or algebraic computation.	Author's verifier (V-receipts); independent script (§10.2).
Exact derived	Exact consequence of displayed identities or of a stated construction.	The derivation given in the text.
Lean-checked	Statement compiled in Lean 4.33.0 at a pinned commit, with an axiom audit; its scope is stated at the claim.	CI record at the pin (Appendix E).
Conditional	Proved under named premises; it is exactly as strong as the weakest premise.	Lean theorem with the premises as hypotheses.
Source declaration	Typed carrier, boundary or partial map retained at its cited source.	Citation only.
Interface	Declared map between carriers or readers.	Declaration only.
Candidate	Next construction, with its test stated.	None yet.

1.2 The governing rule

> **A result carries exactly the strength of its weakest named premise. Nothing crosses a type boundary by renaming.**

Three failures are audited for in every statement: what was *dropped* (a needed item is absent), what was *added* without support (a claim, or a check with no content behind it), and whether the stated *strength* exceeds the premises. Earlier receipts are append-only; a correction is a new receipt, not an edit.

1.3 Coordinate discipline

No word enters this register until four requirements are met:

$$\text{word} \longleftrightarrow \text{typed map} \longleftrightarrow \text{invariant or condition} \longleftrightarrow \text{formal source coordinate.}$$

Each source carries a transmitted text written by a person, the structure of its native form, a stated condition, and a recoverable office. Textual notices, constructions, field equations, mathematical theorems, computational procedures and current source declarations keep distinct record types. Citations keep their source-native lettering; each section opens with a local key for the symbols it uses, and Appendix D reconciles the symbols that share letters.

2 The inherited record

One signature practice of *Homo sapiens* is the deliberate use of enduring marks. A two-dimensional mark holds a noticed relation still: a motion in ordinary life, a relation between people, or a recurring observation of the world. It is a recoverable trace of an earlier occurrence.

In ink, stone, diagrams, archives and binary screens, it lets a later reader reconstruct a prior observation. Lines, tallies, maps, scores, equations and proofs let one reader carry a relation to another and across time. The first observer need not remain present.

This practice became a living lineage of reference, discovery, trial, correction, invention and reuse. Measurement and charting refine fit, make function more predictable, and give time a recoverable record. What works can be carried forward. Failure can teach. The two-dimensional reader serves the living read. A reference enters the lineage when a noticed difference becomes a recognized pattern, receives a fixed orientation and crafted form, and remains transportable, shareable and repairable. When it holds under declared conditions and holds again, it earns trust as usable reference. Its continued use requires that its needed differences remain carried as conditions change. *Forget* names their collapse at a declared reader.

Reader-first order. The historical ledger (Appendix A) remembers who fixed the references. The public form begins with how a relation becomes readable and shareable:

$$\begin{aligned}&\text{contact} \to \text{carried return} \to \text{identity} \to \text{recognition}\\ &\to \text{calibration} \to \text{finite chart} \to \text{update} \to \text{receipt.}\end{aligned}$$

The finite two-dimensional reader comes after contact and calibration. It records an exact invariant relation inside its declared carrier. Field, reader, light, memory and identity keep their own source offices. The source condition of contact is itself ordered: address fixes a state, gate fixes an admitted operation, carrier retains the evolving value, boundary fixes scope, and contact yields a readable record.

Order of this paper. §3 states the reader basis using declared interfaces only. §4–§6 build the exact finite root; nothing in them depends on §3. §7 joins the two. §8–§9 carry the jurisdictional and field-facing layers, with their Lean realizations. §10 gives the receipts and §11 the closing word and the closure statement.

3 The reader basis

This section uses declared interfaces only. No matrix of the finite root is needed to read it; where one is mentioned, the pointer is forward.

3.1 Offices

Local key. c_t is present contact; ι the retained identity criterion; Σ the declared scope; ρ_Σ the calibrated reader; μ_Σ its memory coordinate; $K_{\rho,t}$ its declared knowledge at one occurrence; r_t that occurrence's receipt.

$$\begin{aligned}c_t &:= \text{present typed contact},\\ \mu_\Sigma(t) &:= \text{phase-coordinated retention of the contact relation at scope } \Sigma,\\ K_{\rho,t} &:= \mathrm{Read}_{\rho_\Sigma}\big(c_t, \mu_\Sigma(t), \iota\big), \qquad a \text{ is knowable to } \rho \text{ at } t \iff a \in K_{\rho,t},\\ r_t &:= \text{retained source, calibration and transform, with the residual for one occurrence.}\end{aligned}$$

These are reader offices and declared interfaces (**Source declaration**); they do not enlarge the theorem scope of the finite verifier.

Receipt and memory. The receipt r_t lets another reader reconstruct a stated occurrence. Memory μ_Σ is the phase-coordinated retention of the contact's relation at a declared scope, by which occurrences remain comparable. They are separate offices and neither is typed as the other.

3.2 Recognition and forget

Local key. $\rho : X \to Y$ is a reader, $d_{\mathrm{req}} : X \to D_{\mathrm{req}}$ a required distinction, $\delta_{\mathrm{req}} : Y \to D_{\mathrm{req}}$ a decoder, $x_1, x_2 \in X$ witness states.

Definition 3.1 (Recognition and forget)**.** **Exact derived**The reader ρ *recognizes* d_{req} when $d_{\mathrm{req}} = \delta_{\mathrm{req}} \circ \rho$ for some decoder δ_{req}. The reader *forgets* d_{req} when there exist $x_1, x_2 \in X$ with $\rho(x_1) = \rho(x_2)$ and $d_{\mathrm{req}}(x_1) \neq d_{\mathrm{req}}(x_2)$.

Proposition 3.2. ***Exact derived****If ρ forgets d_{req} then no decoder δ_{req} satisfies $d_{\mathrm{req}} = \delta_{\mathrm{req}} \circ \rho$; if ρ recognizes d_{req} no such pair x_1, x_2 exists. The two conditions are exclusive and, on a given d_{req}, exhaustive.*

Derivation. A decoder satisfying the equation would give $d_{\mathrm{req}}(x_1) = \delta_{\mathrm{req}}(\rho(x_1)) = \delta_{\mathrm{req}}(\rho(x_2)) = d_{\mathrm{req}}(x_2)$, contradicting the witness pair. Conversely, if d_{req} is constant on each fibre of ρ, define $\delta_{\mathrm{req}}(y)$ as that common value for $y \in \rho(X)$, and arbitrarily elsewhere (D_{req} is nonempty when X is). □

Scope recognition is the silent coherence of a sufficiently prepared reader: contact, retained pattern, timing and action arrive as one usable relation before an explicit account is produced,

$$\mathrm{ScopeRecognize}_{\rho,\Sigma}(c_t) := \mathrm{Recognize}_{\rho}\big(c_t, \mu_\Sigma(t), \iota\big) \qquad (\textbf{Source declaration}).$$

The finite root gives one exact instance of two sides that stay distinguishable while their metric relation is invariant under reversal: the mirror satisfies $H^{\mathsf{T}} G H = G$ (Proposition 4.7).

3.3 The twenty-eight words

The coordinate order is: Reader, Identity, Contact, Light, Memory, Recognition, Calibration, Metric, Gravity, Phase, Mirror, Carriage, Wheel, Motion, Scope, Time, Receipt, Return, Lift, Orbit, Worldline, Horizon, Constraint, Gate, Silence, Survival, Forget, Endurance. Each has one sentence here and a typed row (map, condition, source coordinates, status) in Appendix C.

Reader the declared carrier that takes up a relation at a stated resolution.
Identity the retained criterion by which a reader recognizes what must continue through update.
Contact the present typed meeting of carrier, boundary, light, address and stated conditions.
Light present null contact under a metric, with finite null lines in the reader chart.
Memory the phase-coordinated compositional relation that keeps distinct occurrences comparable across a declared scope.
Recognition a required distinction factors through the declared read.
Calibration fitting a read to declared words, instruments, references and residuals.
Metric the signed rule that compares a declared relation; a compatible turn may also supply a positive local read.
Gravity the metric field's stated curvature relation.
Phase a position within an ordered turn.
Mirror orientation reversal preserving the stated metric relation.
Carriage metric-preserving transport along the declared order.
Wheel a finite phase return whose carriage advances one address.

Motion carriage of a declared state under a stated rule.
Scope the declared reference and resolution of a read.
Time ordered carriage measured from a declared scope, with returned phase and retained cycle carry.
Receipt one retained occurrence record with its stated conditions.
Return recurrence at the declared reader.
Lift the retained address after a base read returns.
Orbit an ordered series of returns with retained lifts.
Worldline the declared rail along which an orbit is measured.
Horizon a declared boundary of continuation.
Constraint the condition governing a typed candidate's use.
Gate the typed map assigning admission or fold.
Silence a complete declared read with zero required residual.
Survival the continuation condition carried with admission while the required invariant remains retained through motion.
Forget a required distinction fails to factor through a declared read: one readout carries different required values.
Endurance the closing read: after a half-turn the form survives, the difference remains, the reference returns and the read balances to zero, with nothing erased.

4 The finite root

This section is exact and self-contained. Every statement is a computation on 2×2 matrices over $\mathbb{Q}(\sqrt{5})$; each is given with its derivation, cites the author's receipts *(V-…)*, and is reproduced by the independent script of §10.2.

4.1 The carrier and the three root matrices

Local key. $V = \mathbb{R}^2$ is the reader plane and I its identity map. $\Phi = (1+\sqrt{5})/2$ is the golden ratio, so $\Phi^2 = \Phi + 1$ and $1 - \Phi = -1/\Phi$. Q is the recurrence matrix, J the finite turn, Ω the oriented-pair form.

Definition 4.1 (Root matrices)**.**

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = -J, \qquad \omega(u, v) := u^{\mathsf{T}} \Omega v.$$

Proposition 4.2 (Recurrence)**.** ***Exact derived*** *(H05 source; P05 chart)* $Q^2 = Q + I$ *and* $\det Q = -1$*. With* $\mathcal{F}_0^{\mathrm{Fib}} = 0$*,* $\mathcal{F}_1^{\mathrm{Fib}} = 1$ *and* $\mathcal{F}_{n+1}^{\mathrm{Fib}} = \mathcal{F}_n^{\mathrm{Fib}} + \mathcal{F}_{n-1}^{\mathrm{Fib}}$*, for* $n \geq 1$

$$Q^n = \begin{pmatrix} \mathcal{F}_{n+1}^{\mathrm{Fib}} & \mathcal{F}_n^{\mathrm{Fib}} \\ \mathcal{F}_n^{\mathrm{Fib}} & \mathcal{F}_{n-1}^{\mathrm{Fib}} \end{pmatrix}, \qquad Q^n \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathcal{F}_{n+1}^{\mathrm{Fib}} \\ \mathcal{F}_n^{\mathrm{Fib}} \end{pmatrix}.$$

Derivation. $Q^2 = \left(\begin{smallmatrix} 2 & 1 \\ 1 & 1 \end{smallmatrix}\right) = Q + I$. The formula holds at $n = 1$, and multiplying it by Q on the right gives the same form at $n + 1$ by the recurrence. □

Proposition 4.3 (Dual-form return)**.** ***Exact finite*** *(V-I1; P16)* $Q^{\mathsf{T}} \Omega Q = -\Omega$ *and* $(Q^2)^{\mathsf{T}} \Omega Q^2 = \Omega$*.*

Derivation. For every 2×2 matrix M, $M^{\mathsf{T}} \Omega M = (\det M)\, \Omega$. Here $\det Q = -1$ and $\det Q^2 = 1$. □

4.2 The reader metric

Definition 4.4 (Reader metric)**.** $G := [Q, \Omega] = Q\Omega - \Omega Q$. The signed read on V is $g_G(v, w) := v^{\mathsf{T}} G w$.

Theorem 4.5 (Metric chart)**.** ***Exact finite*** *(V-A1–A5, V-H4; P05)*

$$G = [J, Q] = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}, \qquad G = G^{\mathsf{T}}, \qquad G^2 = 5I, \qquad \det G = -5, \qquad G^{-1} = \tfrac{1}{5} G.$$

The eigenvalues of G are $\pm\sqrt{5}$, so its signature is $(1, 1)$, and $\sqrt{5} = \sqrt{-\det G}$. With $z = (x, y)^{\mathsf{T}}$,

$$z^{\mathsf{T}} G z = -2x^2 + 2xy + 2y^2 = -2(x - \Phi y)\big(x - (1 - \Phi)y\big), \qquad \mathcal{N}_G = \mathbb{R}(\Phi, 1) \cup \mathbb{R}(1 - \Phi, 1).$$

Moreover $Q^{\mathsf{T}} G Q = -G$ and $(Q^2)^{\mathsf{T}} G Q^2 = G$.

Derivation. $Q\Omega = \left(\begin{smallmatrix} -1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$ and $\Omega Q = \left(\begin{smallmatrix} 1 & 0 \\ -1 & -1 \end{smallmatrix}\right)$; their difference is G. Since $\Omega = -J$, $[Q, \Omega] = [J, Q]$. Squaring gives $5I$. The factorization follows from $\Phi + (1 - \Phi) = 1$ and $\Phi(1 - \Phi) = -1$: the product is $-2(x^2 - xy - y^2)$. For the last claim, Q is symmetric and $Q^2 = Q + I$, so $Q^{\mathsf{T}} G Q = Q^2 \Omega Q - Q \Omega Q^2 = (Q + I)\Omega Q - Q\Omega(Q + I) = \Omega Q - Q\Omega = -G$. □

4.3 The mirror

Definition 4.6 (Mirror)**.** $H := G/\sqrt{5}$. Its entries, and those of S and L_{ax} below, lie in $\mathbb{Q}(\sqrt{5})$.

Proposition 4.7 (Mirror)**.** ***Exact finite*** *(V-B2, V-E1–E7; P05) $H = H^{\mathsf{T}}$, $H^2 = I$, and $H^{\mathsf{T}} G H = G$.*

Derivation. $H^2 = G^2/5 = I$ and $H^{\mathsf{T}} G H = G^3/5 = G\,(G^2/5) = G$, using $G = G^{\mathsf{T}}$ and Theorem 4.5. □

4.4 The turn, the phase family and the local i-fold

Theorem 4.8 (Turn)**.** ***Exact finite*** *(V-N1–N3; P05, P17) (i) $J^2 = -I$ and $J^{\mathsf{T}} J = I$. (ii) $J^{\mathsf{T}} G J = -G$: one turn reverses the metric sign. (iii) J exchanges the two null lines: $J(\Phi, 1)^{\mathsf{T}} = \Phi\,(1 - \Phi, 1)^{\mathsf{T}}$ and $J(1 - \Phi, 1)^{\mathsf{T}} = (1 - \Phi)(\Phi, 1)^{\mathsf{T}}$. (iv) $HJH = J^{-1} = -J$.*

Derivation. (i) is direct. (ii) $GJ = \left(\begin{smallmatrix} 1 & 2 \\ 2 & -1 \end{smallmatrix}\right)$ and $J^{\mathsf{T}}(GJ) = \left(\begin{smallmatrix} 2 & -1 \\ -1 & -2 \end{smallmatrix}\right) = -G$. (iii) $J(\Phi, 1)^{\mathsf{T}} = (-1, \Phi)^{\mathsf{T}}$ and $\Phi(1 - \Phi) = -1$; the second identity is the same computation with $\Phi \leftrightarrow 1 - \Phi$. (iv) From (ii) and $J^{\mathsf{T}} = -J$ we get $JGJ = G$, hence $JG = GJ^{-1} = -GJ$ and $GJG = -JG^2 = -5J$; so $HJH = GJG/5 = -J$. □

Two consequences are used throughout. With $U_\alpha := e^{\alpha J} = \cos\alpha\, I + \sin\alpha\, J$ one has $J = U_{\pi/2}$, $U_\pi = -I$, $U_{2\pi} = I$ and $H U_\alpha H = U_{-\alpha}$ **Exact finite** *(V-B1, B3, H2–H3, H5, H9, N6).* Under $(x, y) \leftrightarrow x + iy$ the map J is multiplication by i.

Proposition 4.9 (Local i-fold)**.** ***Exact finite*** *(V-I2; P17) $g_J(u, v) := \omega(u, Jv) = u^{\mathsf{T}} \Omega J v = u^{\mathsf{T}} v$, so $g_J(u, u) = \|u\|^2 \geq 0$.*

Derivation. $\Omega J = -J^2 = I$. □

The positive local read g_J and the signed read g_G share the carrier V and keep different conditions: g_J is generated by the oriented relation and its compatible turn, g_G by transport and orientation order.

4.5 The carriage

Definition 4.10 (Carriage)**.** $S := HJ$ and $L_s := e^{sS}$ for $s \in \mathbb{R}$; $L_{\mathrm{ax}} := L_{\log \Phi}$.

Theorem 4.11 (Carriage)**.** ***Exact finite*** *(V-B4–B6, V-H1, V-E7, V-N4, V-N7; P05)*

(i) $S = (2Q - I)/\sqrt{5}$, $S = S^{\mathsf{T}}$, $S^2 = I$.
(ii) $L_s = \cosh s\, I + \sinh s\, S$, $L_t L_s = L_{t+s}$, $\det L_s = 1$, $\operatorname{tr} L_s = 2\cosh s$.
(iii) $L_s^{\mathsf{T}} G L_s = G$: *carriage preserves the metric.*
(iv) $H L_s H = L_{-s}$: *the mirror reverses the order of carriage.*
(v) $L_{\mathrm{ax}} = (Q + 2I)/\sqrt{5}$, $L_{\mathrm{ax}} = L_{\mathrm{ax}}^{\mathsf{T}}$, $L_{\mathrm{ax}}^2 = Q^2$, $L_{\mathrm{ax}}^{\mathsf{T}} G L_{\mathrm{ax}} = G$, $H L_{\mathrm{ax}} H = L_{\mathrm{ax}}^{-1}$.

Derivation. (i) $S = HJ = GJ/\sqrt{5}$ and $GJ = \left(\begin{smallmatrix} 1 & 2 \\ 2 & -1 \end{smallmatrix}\right) = 2Q - I$, which is symmetric. Then $S^2 = (GJG)J/5 = -J^2 = I$, using $GJG = -5J$ from the proof of Theorem 4.8. (ii) follows from $S^2 = I$ by the exponential series. (iii) S and G anticommute: $GS = G\,HJ = GGJ/\sqrt{5} = \sqrt{5}J$ and $SG = HJG = GJG/\sqrt{5} = -\sqrt{5}J$. Hence $GL_s = L_{-s}G$, and since L_s is symmetric, $L_s^{\mathsf{T}} G L_s = L_s G L_s = L_s L_{-s} G = G$. (iv) $HSH = H\,HJ\,H = JH = -HJ = -S$ by Theorem 4.8(iv), so $HL_sH = L_{-s}$. (v) $\cosh \log \Phi = (\Phi + \Phi^{-1})/2 = \sqrt{5}/2$ and $\sinh \log \Phi = (\Phi - \Phi^{-1})/2 = \frac{1}{2}$, so $L_{\mathrm{ax}} = \frac{\sqrt{5}}{2} I + \frac{2Q - I}{2\sqrt{5}} = \frac{Q + 2I}{\sqrt{5}}$. Then $L_{\mathrm{ax}}^2 = (Q^2 + 4Q + 4I)/5 = Q + I = Q^2$, and the remaining identities are (iii), (iv) at $s = \log \Phi$. □

Proposition 4.12 (The carriage is never orthogonal)**.** ***Exact derived*** *(finite guard V-E8, $1 \le n \le 8$) For every $n \ge 1$, $(L_{\mathrm{ax}}^n)^{\mathsf{T}} L_{\mathrm{ax}}^n = Q^{2n} \neq I$.*

Derivation. L_{ax} is symmetric, so $(L_{\mathrm{ax}}^n)^{\mathsf{T}} L_{\mathrm{ax}}^n = L_{\mathrm{ax}}^{2n} = Q^{2n}$ by Theorem 4.11(v). By Proposition 4.2 the off-diagonal entry of Q^{2n} is $\mathcal{F}_{2n}^{\mathrm{Fib}} \ge 1$. □

The author's verifier records only the finite range $1 \le n \le 8$ and declines to extend it; the derivation above extends it to all $n \ge 1$.

4.6 The sign character and the reflection–turn group

Definition 4.13 (Carriage group)**.** $\Gamma_{\mathrm{rot}} := \langle J, H, L_{\mathrm{ax}} \rangle \subset GL_2(\mathbb{Q}(\sqrt{5}))$, and $D_4 := \langle J, H \rangle$.

Theorem 4.14 (Sign character)**.** ***Exact finite/Exact derived*** *(V-E1–E7; V-E9 finite guard; P05) For every $\gamma \in \Gamma_{\mathrm{rot}}$, $\gamma^{\mathsf{T}} G \gamma = \chi(\gamma)\, G$ with $\chi(\gamma) \in \{+1, -1\}$, and $\chi : \Gamma_{\mathrm{rot}} \to \{\pm 1\}$ is a homomorphism with*

$$\chi(J) = -1, \qquad \chi(J^2) = \chi(H) = \chi(L_{\mathrm{ax}}) = +1.$$

In particular $H^{\mathsf{T}} G H = G$, which Proposition 4.7 derives directly.

Derivation. The three generators satisfy the law with the stated values by Theorem 4.8(ii), Proposition 4.7 and Theorem 4.11(iii). If γ_1, γ_2 satisfy it, then $(\gamma_1\gamma_2)^{\mathsf{T}} G \gamma_1\gamma_2 = \gamma_2^{\mathsf{T}}(\gamma_1^{\mathsf{T}} G \gamma_1)\gamma_2 = \chi(\gamma_1)\gamma_2^{\mathsf{T}} G \gamma_2 = \chi(\gamma_1)\chi(\gamma_2) G$. G is invertible, so $\chi(\gamma)$ is determined by γ and is therefore multiplicative. □

The verifier runs the multiplication law over all twenty-seven words of length three in J, H, L_{ax} *(V-E9)*. The sign resolves phase modulo two; the four-address phase coordinate of §5 carries the remaining distinction.

Proposition 4.15 (Reflection–turn group)**.** ***Exact derived*** *(V-N6) $D_4 = \langle J, H \rangle$ has order eight and is dihedral: J has order four, $H^2 = I$, $HJH = J^{-1}$, and $H \notin \langle J \rangle$.*

Derivation. $\langle J \rangle = \{I, J, -I, -J\}$ consists of rotations ($\det = 1$), while $\det H = \det G/(\sqrt{5})^2 = -1$. With $HJH = J^{-1}$ the group is $\langle J \rangle \cup H\langle J \rangle$. □

5 Return, rotation and time

Local key. $z \in V$ is the carried address and $\varphi \in \mathbb{Z}/4$ its phase address. F_{esc} is the four-step wheel map, L_{ax} its carried step, γ a declared chart transformation, G the reader metric and $\varepsilon_{\mathrm{mech}}$ the paired phase-and-metric residual.

5.1 The wheel

The phase wheel and the axial carriage have different offices:

$$F_{\mathrm{esc}}(\varphi, z) = \begin{cases} (\varphi + 1, z), & \varphi = 0, 1, 2, \\ (0, L_{\mathrm{ax}} z), & \varphi = 3, \end{cases} \qquad F_{\mathrm{esc}}^4(\varphi, z) = (\varphi, L_{\mathrm{ax}} z) \ \text{ for every } \varphi \in \mathbb{Z}/4.$$

Proposition 5.1 (Wheel return)**.** ***Exact finite*** *(V-B7, V-H7, V-N5; P06) After four steps the phase address is returned and the carried address has advanced by exactly one L_{ax}.*

Derivation. From $\varphi = 0$: $(0, z) \to (1, z) \to (2, z) \to (3, z) \to (0, L_{\mathrm{ax}} z)$. From any other φ the single carry occurs at the unique step where the phase passes from 3 to 0, so the same end state $(\varphi, L_{\mathrm{ax}} z)$ results. □

This is the mechanical statement of the register: *return admits relational difference.* The orbit $z_n = L_{\mathrm{ax}}^n z_0$ satisfies $F_{\mathrm{esc}}^{4n}(\varphi, z_0) = (\varphi, z_n)$.

5.2 The residual of one rotation

Definition 5.2 (Mechanical residual)**.** $\varepsilon_{\mathrm{mech}}(\varphi, z; \gamma) := \big(\varepsilon_\varphi(\varphi, z),\, \varepsilon_G(\gamma; z)\big)$ with $\varepsilon_\varphi(\varphi, z) = \mathrm{pr}_\varphi F_{\mathrm{esc}}^4(\varphi, z) - \varphi$ and $\varepsilon_G(\gamma; z) = (\gamma z)^{\mathsf{T}} G(\gamma z) - z^{\mathsf{T}} G z$.

Proposition 5.3. ***Exact finite*** *(V-H6–H7; P05, P06) $\varepsilon_\varphi = 0$; $\varepsilon_G(J^n; z) = \big((-1)^n - 1\big) z^{\mathsf{T}} G z$; $\varepsilon_G(H; z) = \varepsilon_G(L_{\mathrm{ax}}^n; z) = 0$.*

Derivation. Proposition 5.1, Theorem 4.8(ii), Proposition 4.7 and Theorem 4.11(iii). □

Recognition of one complete rotation of the wheel is $\varepsilon_{\mathrm{mech}} = 0$: its phase address is returned and its metric comparison is retained. The half-turn J^2, the metric mirror and the axial carriage retain the metric read; a single turn does not.

5.3 Return operators

Operator	Exact law	Office	Status
H	$H^2 = I$	Mirror: reciprocal orientation.	**Exact finite**
$i_{\mathrm{turn}} := J$	$i_{\mathrm{turn}}^2 = -I$	Turn: quarter-phase action.	**Exact finite**
$H i_{\mathrm{turn}} H$	$H i_{\mathrm{turn}} H = i_{\mathrm{turn}}^{-1} = -i_{\mathrm{turn}}$	Returned turn: reversal inside the same relation.	**Exact finite**
L_s	$H L_s H = L_{-s}$	Carriage and reciprocal carriage.	**Exact derived**
F_{esc}	$F_{\mathrm{esc}}^4(\varphi, z) = (\varphi, L_{\mathrm{ax}} z)$	Wheel return: phase repeats; carried address advances.	**Exact finite**

The forward and return directions are dual directions of one declared return relation. Neither supplies an external ground for the other. i_{turn} keeps its subscript because bare i is the scope point of §5.4.

5.4 The rail, the axial coordinate and time

Local key. $\mathbb{H}$ is the upper half-plane, on which $L = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ of determinant 1 acts by $w \mapsto (aw+b)/(cw+d)$. The declared rail $\{\delta = 0\} \subset \mathbb{H}$ is the axis of L_{ax}; $i \in \mathbb{H}$ is its scope point; Λ is signed hyperbolic arclength from i along the rail, oriented toward Φ. $T = 2\log\Phi$ is one axial period; W is the recurring circle read; ℓ_0 a declared metric scale; τ a proper-time coordinate; ρ_t, β_t the phase reader and its angle at parameter t.

Proposition 5.4 (The rail)**.** ***Exact derived*** *(V-F1–F7; P05) (i) L_{ax} fixes exactly the two boundary points Φ and $1-\Phi$, the roots of $w^2 - w - 1$. The rail is the geodesic joining them: the semicircle of centre $\frac{1}{2}$ and radius $\frac{\sqrt{5}}{2}$. It contains i and is invariant under every L_s. (ii) $\operatorname{tr} L_s = 2\cosh s$, so the metric translation length is $\ell_{\mathrm{ax}}(L_s) = 2\operatorname{arccosh}(\operatorname{tr} L_s/2) = 2|s|$. (iii) On the parameterized rail $z(s) = L_s i$, set $\Lambda(z(s)) = 2s$. Then $\Lambda(L_t z) = \Lambda(z) + 2t$ on the rail, and $\Lambda(L_{\mathrm{ax}} z) = \Lambda(z) + T$ with $T = 2\log\Phi$.*

Derivation. Since $L_{\mathrm{ax}} = (Q + 2I)/\sqrt{5}$ acts as $w \mapsto (3w+1)/(w+2)$, its fixed points satisfy $w^2 - w - 1 = 0$. Every $L_s = \cosh s\, I + \sinh s\, S$ shares the eigenvectors of S, so all share these fixed points and preserve the geodesic joining them. $|i - \frac{1}{2}|^2 = \frac{5}{4}$, so i lies on that semicircle. Part (ii) is the trace formula for translation length of an element of $SL_2(\mathbb{R})$. For (iii), the group law $L_t L_s = L_{t+s}$ (Theorem 4.11(ii)) turns L_t into the shift $s \mapsto s + t$ on the parameter, and $\Lambda = 2s$. As a check, $L_{\mathrm{ax}} i = \frac{3i+1}{i+2} = 1 + i$ lies on the rail and $\cosh d(i, 1+i) = 1 + \frac{1}{2} = \frac{3}{2} = \cosh(2\log\Phi)$. □

The signed coordinate Λ and the nonnegative translation length ℓ_{ax} are distinct coordinates of the same carriage. The orbit $z_n = L_{\mathrm{ax}}^n z_0$ is the sample at $s = n\log\Phi$; the rail together with this coordinate carries the declared worldline.

Circle read, metric scale and phase reader. The circle read is $W(\Lambda) = \exp(2\pi i \Lambda/T)$, so $W(\Lambda + T) = W(\Lambda)$: Λ carries ordered address and W reads recurring phase. Under a declared axial metric scale ℓ_0 (**Source declaration**),

$$ds^2_{\mathrm{ax}} = -\ell_0^2\, d\Lambda^2, \qquad d\tau = \ell_0\, d\Lambda, \qquad \rho_t(\Lambda) = \exp\Big(\tfrac{t\Lambda}{2} J\Big), \qquad \frac{d\beta_t}{d\tau} = \frac{t}{2\ell_0}.$$

The metric measures the ordered carriage through which phase advances.

5.5 Three return normal forms

Local key. p_{ax} is the axial quotient map and $\bar{\alpha}_{\mathrm{ret}}$ its descended harmonic one-form with $p_{\mathrm{ax}}^* \bar{\alpha}_{\mathrm{ret}} = d\Lambda$; core is a positively oriented core cycle. z_{pno} is the piano phase read, W_{pno} its lifted key address, $\mathcal{H}_{\mathrm{pno}}$ its retained history. Source-native $p, z_w, W_k, \mathcal{H}_k$ remain at citation.

The same return grammar occurs on three separately typed carriers:

Carrier	Returned read	Continuing coordinate	Retained relation	Status
finite four-phase wheel	φ	$L_{\mathrm{ax}} z$	$F^4_{\mathrm{esc}}(\varphi, z) = (\varphi, L_{\mathrm{ax}} z)$	**Exact finite**
axial quotient	$p_{\mathrm{ax}}(L_{\mathrm{ax}} z) = p_{\mathrm{ax}}(z)$	$\Lambda(L_{\mathrm{ax}} z) = \Lambda(z) + T$	$\oint_{\mathrm{core}} \bar{\alpha}_{\mathrm{ret}} = T$	**Exact derived**, **Source declaration**
twelve-key temperament (P19)	$z_{\mathrm{pno}}(k+12) = z_{\mathrm{pno}}(k)$	$W_{\mathrm{pno}}(k+12) = W_{\mathrm{pno}}(k) + 1$	$\mathcal{H}_{\mathrm{pno}}(k) \prec \mathcal{H}_{\mathrm{pno}}(k+12)$	**Source declaration**

Each row fixes the same structural relation at its own scale: a phase read closes, an address advances, and a retained relation preserves the traversal.

6 Action, charts and addresses

6.1 The finite action and its conserved quantity

Local key. $z(s) \in V$ is the carried reader state and $\dot{z}$ its derivative in the carriage parameter s. Ω is the oriented-pair form, H the mirror, $S = HJ$ the carriage generator, $\mathcal{L}_{\mathrm{ax}}$ the finite action and $\mathcal{H}_{\mathrm{ax}}$ its conserved quadratic quantity.

Because $\Omega S = -H$, the root carries a first-order action

$$\mathcal{L}_{\mathrm{ax}}(z, \dot{z}) = \tfrac{1}{2}\, z^{\mathsf{T}} \Omega \dot{z} + \tfrac{1}{2}\, z^{\mathsf{T}} H z, \qquad \mathcal{H}_{\mathrm{ax}}(z) = -\tfrac{1}{2}\, z^{\mathsf{T}} H z.$$

Theorem 6.1 (Finite action)**.** ***Exact finite*** *(V-L1–L4; P12) (i) $\Omega S = -H$. (ii) The Euler–Lagrange equation of $\mathcal{L}_{\mathrm{ax}}$ is $\Omega\dot{z} + Hz = 0$, equivalently $\dot{z} = Sz$, equivalently $z(s) = L_s z(0)$. (iii) $S^{\mathsf{T}}H + HS = 0$, and $\mathcal{H}_{\mathrm{ax}}(z(s))$ is constant along every solution.*

Derivation. (i) By Theorem 4.8(iv) $JH = -HJ$, so $JHJ = -HJ^2 = H$ and $\Omega S = -J(HJ) = -H$. (ii) With $\Omega^{\mathsf{T}} = -\Omega$ and fixed endpoints, $\delta \int \mathcal{L}_{\mathrm{ax}}\, ds = \int \delta z^{\mathsf{T}} (\Omega\dot{z} + Hz)\, ds$. Since $\Omega^{-1} = -\Omega$ and $\Omega H = -JH = HJ = S$, $\Omega\dot{z} + Hz = 0$ gives $\dot{z} = -\Omega^{-1}Hz = \Omega H z = Sz$, whose solution is $e^{sS}z(0)$. (iii) S is symmetric, so $S^{\mathsf{T}}H + HS = HJH + H\,HJ = -J + J = 0$. Also $HS = J$ is antisymmetric, so $\frac{d}{ds}\mathcal{H}_{\mathrm{ax}} = -z^{\mathsf{T}}H\dot{z} = -z^{\mathsf{T}}HSz = -z^{\mathsf{T}}Jz = 0$. □

This action has the finite reader as its carrier. The field action of §9 keeps its own carrier and its stated regular branch.

6.2 The projective and unitary chart

Local key. M_{pr} is the projective reflection, K_{pr} its positive chart form, Z_{ch} the complex chart map, U_{pr} its quarter-turn, A_{ch} the real adapter and L_{pr} the transported axial carriage. The source retains M, K, P as native labels; the local labels isolate this chart from the manifold $\mathcal{M}$, the analytic trace weight K_{tr} and the source labels P01–P19.

$$M_{\mathrm{pr}} = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}, \quad K_{\mathrm{pr}} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}, \quad Z_{\mathrm{ch}} = \begin{pmatrix} 1 & i \\ 0 & i \end{pmatrix}, \quad U_{\mathrm{pr}} = iM_{\mathrm{pr}}.$$

Theorem 6.2 (Projective chart)**.** ***Exact finite*** *(V-C1–C6; P09) $M_{\mathrm{pr}}^2 = I$, $\det M_{\mathrm{pr}} = -1$; $U_{\mathrm{pr}}^2 = -I$, $U_{\mathrm{pr}}^4 = I$; $U_{\mathrm{pr}} = Z_{\mathrm{ch}} J Z_{\mathrm{ch}}^{-1}$; $K_{\mathrm{pr}} = (Z_{\mathrm{ch}}^{-1})^{\dagger} Z_{\mathrm{ch}}^{-1} \succ 0$; $J^{\dagger}J = I$; $U_{\mathrm{pr}}^{\dagger} K_{\mathrm{pr}} U_{\mathrm{pr}} = K_{\mathrm{pr}}$.*

Theorem 6.3 (Real adapter)**.** ***Exact finite*** *(V-D1–D5; P09) With*

$$A_{\mathrm{ch}} = \begin{pmatrix} 1 & 2+\sqrt{5} \\ 0 & \sqrt{5} \end{pmatrix} : \qquad A_{\mathrm{ch}} H A_{\mathrm{ch}}^{-1} = M_{\mathrm{pr}},$$

$$L_{\mathrm{pr}} := A_{\mathrm{ch}} L_{\mathrm{ax}} A_{\mathrm{ch}}^{-1} = \begin{pmatrix} 1+\sqrt{5} & -2-\sqrt{5} \\ 1 & -1 \end{pmatrix}, \qquad M_{\mathrm{pr}} L_{\mathrm{pr}} M_{\mathrm{pr}} = L_{\mathrm{pr}}^{-1}, \quad \det L_{\mathrm{pr}} = 1.$$

The adapter family is $\{A' : A'H = M_{\mathrm{pr}}A'\} = \{(xI + yM_{\mathrm{pr}})A_{\mathrm{ch}} : x, y \in \mathbb{R}\}$, a two-dimensional space.

Both are direct matrix computations over $\mathbb{Q}(\sqrt{5}, i)$. The family claim holds because $M_{\mathrm{pr}}(xI + yM_{\mathrm{pr}}) = (xI + yM_{\mathrm{pr}})M_{\mathrm{pr}}$, and because the solution space of $A'H = M_{\mathrm{pr}}A'$ is two-dimensional for two reflections of the same eigenvalue type.

6.3 Metric address and the finite factor profile

Local key. $x = a + b\Phi \in \mathbb{Z}[\Phi]$ is a quadratic-integer address, $z_x = (b, a)^{\mathsf{T}}$ its reader-plane coordinate, N_{alg} its signed algebraic norm, and Fac_G its declared prime-factor profile.

Proposition 6.4 (Norm address)**.** ***Exact finite*** *(V-I3; P15)*

$$N_{\mathrm{alg}}(a + b\Phi) = a^2 + ab - b^2, \qquad z_x^{\mathsf{T}} G z_x = 2N_{\mathrm{alg}}(x),$$
$$N_{\mathrm{alg}}(\Phi x) = -N_{\mathrm{alg}}(x), \qquad N_{\mathrm{alg}}(\Phi^2 x) = N_{\mathrm{alg}}(x).$$

Derivation. N_{alg} is the product of the two conjugates, $(a + b\Phi)(a + b(1 - \Phi)) = a^2 + ab - b^2$. With $z_x = (b, a)^{\mathsf{T}}$, $z_x^{\mathsf{T}} G z_x = -2b^2 + 2ab + 2a^2$. Since $\Phi(a + b\Phi) = b + (a + b)\Phi$, $N_{\mathrm{alg}}(\Phi x) = b^2 + b(a + b) - (a + b)^2 = -(a^2 + ab - b^2)$. □

For $x \neq 0$ define $\mathrm{Fac}_G(x) := \big(v_p(|N_{\mathrm{alg}}(x)|)\big)_{p \text{ prime}} \in \bigoplus_p \mathbb{N}$, the prime profile of the absolute norm. Because the absolute norm is carried through the Φ-step, $\mathrm{Fac}_G(\Phi x) = \mathrm{Fac}_G(x)$ **Exact derived**. Fac_G is a declared arithmetic reader: it may enter a meaning or function packet only through a stated reader, comparison and use condition.

Proposition 6.5 (Four-wheel finite address)**.** ***Exact finite*** *(V-I4; P18)* $\mathcal{F}_9^{\mathrm{Fib}} = 34$, $\mathcal{L}_5^{\mathrm{Luc}} = 11$, *and* $137 = 1 + 4\mathcal{F}_9^{\mathrm{Fib}} = (\mathcal{L}_5^{\mathrm{Luc}})^2 + 4^2$. *In* $\mathbb{F}_{137}^{\times}$*:* $4^{17} \equiv -1 \pmod{137}$, $\mathrm{ord}_{137}(4) = 34$, *and* $\mathbb{F}_{137}^{\times}/\langle 4 \rangle \cong C_4$. *Also* $548 = 4 \cdot 137$ *and* $\mathbb{Z}/548\mathbb{Z} \cong \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/137\mathbb{Z}$.

Derivation. $1 + 136 = 137$ and $121 + 16 = 137$. $4^{17} \equiv 136 \equiv -1$, so $4^{34} \equiv 1$; the order divides 34 and exceeds 2 (since $4^2 = 16 \neq 1$) and 17 (since $4^{17} \neq 1$), so it is 34. $\mathbb{F}_{137}^{\times}$ is cyclic of order $136 = 4 \cdot 34$, so the quotient by the subgroup of order 34 is cyclic of order 4. $\gcd(4, 137) = 1$ gives the Chinese-remainder isomorphism. □

This is a finite address closure: a four-phase quotient carries an order-34 multiplicative transport. Any empirical calibration retains its own carrier and conditions.

Arithmetic fold interface. **Candidate** $\mathcal{T}_{\mathrm{arith}} : \mathcal{K}_{\mathrm{arith}} \to \mathcal{P}_i$ names a proposed arithmetic-to-positive-read map between declared carriers, with $\mathcal{Q}_{\mathrm{arith}}(f) = \Omega_{\mathcal{P}_i}(\mathcal{T}_{\mathrm{arith}} f, J_i \mathcal{T}_{\mathrm{arith}} f) = \|\mathcal{T}_{\mathrm{arith}} f\|^2$, where $\mathcal{Q}_{\mathrm{arith}}$ is a source-declared quadratic form distinct from Q. Its domain, criterion class, map and equality conditions remain to be declared and checked at source. The finite local read g_J is already exact (Proposition 4.9); this line preserves the separate construction an arithmetic extension requires.

6.4 The analytic phase reader

Local key. ζ, ξ retain Riemann's analytic source functions; $\Xi(t) = \xi(\frac{1}{2} + it)$ is their completed scalar read. $\lambda = v + kT$ separates the local phase cell v from the ordered cell count k; U_t reads local phase, $\mathcal{H}_t$ carries it from cell to cell, and K_{tr} is the source-retained trace weight.

Riemann's 1859 source (H14) carries ζ and the completed ξ. The cell decomposition, phase reader and trace construction below are a present source declaration **Source declaration** with their own

carrier. With $U_t(v) = U_{tv/2}$,

$$\lambda = v + kT,\ \ 0 \le v < T,\ k \in \mathbb{Z}_{\ge 0}; \qquad \rho_t(v + kT) = U_t(v)\,\mathcal{H}_t^k,\ \ \mathcal{H}_t = \exp(t \log \Phi\, J);$$
$$\Xi(t) = \sum_{k=0}^{\infty} \int_0^T K_{\mathrm{tr}}(v + kT)\, \tfrac{1}{2}\, \mathrm{tr}\big(U_t(v)\mathcal{H}_t^k\big)\, dv.$$

The first factorization is exact: $\exp\big(\frac{t}{2}(v + kT)J\big) = \exp\big(\frac{tv}{2}J\big)\exp(kt \log \Phi\, J)$ since $T = 2 \log \Phi$ **Exact derived**. A finite phase closure after m axial cells satisfies $\mathcal{H}_t^m = I \iff mt \log \Phi \in 2\pi\mathbb{Z}$ **Exact derived**. The trace weight K_{tr} belongs to its source; the infinite k-sum carries unbounded lift while the local cell remains a returning phase read.

Evenness $\Xi(-t) = \Xi(t)$ is a symmetry of the completed scalar read. It is not a time evolution. Evaluating $\Xi(t)$ is an ordered process on its own carrier (an address t, an algorithm state, a stated method and a stated precision); the static relation and the dynamic computation meet at the evaluation interface $(t, \text{method}, \text{precision}) \mapsto \Xi(t)$.

7 The reader on the root

§3 fixed the reader offices as declared interfaces; §4–§6 fixed the exact root. This section states where they meet and what stays unproved at the join.

7.1 Contact record and normal form

Local key. Π forms a contact record; e_Π is its metric carrier; ℓ is light; w is ordered address; r is the appended receipt. U_{rec} is a declared local update, ρ_{rec} its reader at a stated resolution, and L its ordered outcome ledger.

The contact record retains carrier, relation, address and occurrence together:

$$\Pi(e_\Pi, \ell) = c, \qquad c = (e_\Pi, \ell, w, r). \qquad (\textbf{Source declaration})$$

A second source record (P15) gives the carried contact normal form. With $B_{\mathrm{rec}} := Q^2$, $U_{\mathrm{rec}} : V \times E \to V$, $\rho_{\mathrm{rec}} : V \to Y$ and $\mathsf{L} \in Y^*$,

$$\mathcal{C}_{\rho_{\mathrm{rec}}}(z, e, \mathsf{L}) = \Big(U_{\mathrm{rec}}(B_{\mathrm{rec}} z, e),\ \mathrm{snoc}\big(\mathsf{L}, \rho_{\mathrm{rec}}\, U_{\mathrm{rec}}(B_{\mathrm{rec}} z, e)\big)\Big). \qquad (\textbf{Source declaration})$$

B_{rec} carries reversible transport; ρ_{rec} fixes the reader resolution; L retains the order of reader outcomes. The root supplies one exact fact about B_{rec}: Q^2 is invertible and preserves both ω and G (Proposition 4.3, Theorem 4.5) **Exact derived**. The construction supplies the historical office of receipt without retyping memory: r reconstructs a stated occurrence, while μ_Σ retains the relation by which occurrences remain comparable.

The field-to-reader adapter. The declared transport from the field carrier to the rail $\{\delta = 0\} \subset \mathbb{H}$ of §5.4 is

$$\chi_\Pi : e_\Pi \longrightarrow \{\delta = 0\} \subset \mathbb{H}, \qquad \chi_\Pi \circ L_e = L_{\mathrm{ax}} \circ \chi_\Pi.$$

Candidate It becomes active with a declared field transport L_e, a calibration and a carrier-to-rail map; none is supplied at source.

7.2 Compositional memory

Local key. $\mathcal{P}$ and $\mathcal{R}$ are declared path-or-word categories, id_x is the identity path at x, and MEMORY is the source-declared functor between them. MR is a Memory Return datum; its subscripts prevent collisions with the packet, reader, residual and receipt offices elsewhere.

$$\mathsf{MEMORY} : \mathcal{P} \to \mathcal{R}, \qquad \mathsf{MEMORY}(\gamma_2 \circ \gamma_1) = \mathsf{MEMORY}(\gamma_2) \circ \mathsf{MEMORY}(\gamma_1),$$
$$\mathsf{MEMORY}(\mathrm{id}_x) = \mathrm{id}_{\mathsf{MEMORY}(x)}. \qquad (\textbf{Source declaration};\ \mathrm{H34})$$

The source-normal datum makes its fields explicit:

$$\mathrm{MR} = \big(r,\, x,\, p_{\mathrm{lift}},\, \gamma,\, \tilde{\gamma},\, \varepsilon_{\mathrm{deck}},\, \eta_{\mathrm{act}},\, \rho_{\mathrm{rep}},\, D_{\rho_{\mathrm{rep}}}(\varepsilon_{\mathrm{deck}}),\, R_{\mathrm{MR}}\big), \qquad D_{\rho_{\mathrm{rep}}}(g) = \rho_{\mathrm{rep}}(g) - I.$$

MR retains a return reader, a lift, an endpoint defect, a represented difference and a reconstructible occurrence record. MEMORY answers how occurrences remain comparable; R_{MR} answers which occurrence was retained. Their offices remain separate.

7.3 Interlocking structural compression

Proposition 7.1 (Four relations in one chart)**.** ***Exact finite*** *(V-I1, V-I2, V-B2; P05, P16, P17) In the finite chart:* $J^2 = -I$, $H^2 = I$, $HJH = -J$; $g_J(u,v) = \omega(u, Jv) = u^{\mathsf{T}} v$; $J^{\mathsf{T}}\Omega J = \Omega$ *and* $J^{\mathsf{T}} G J = -G$; $Q^{\mathsf{T}}\Omega Q = -\Omega$, $Q^{\mathsf{T}} G Q = -G$; $(Q^2)^{\mathsf{T}}\Omega Q^2 = \Omega$, $(Q^2)^{\mathsf{T}} G Q^2 = G$.

Derivation. Theorems 4.5 and 4.8, Propositions 4.3 and 4.9; $J^{\mathsf{T}}\Omega J = \Omega$ because $\det J = 1$. □

One recurrence step Q reverses both signed forms (ω and G) and Q^2 restores them. The turn J preserves ω and reverses G only. The positive local read g_J, the signed metric read g_G and the return pair (J, H) therefore occupy one finite chart with distinct jobs. The contact operator carries an ordered update and reader outcome; MEMORY carries comparison through composition; the receipt retains a reconstructible occurrence.

7.4 The read–write relation

Local key. i_{turn} is the finite turn, H its mirror, Φ_{rd} a declared read of e-carriage, Ψ_{wr} a declared write of g-memory. $\tilde{E}_e$ and $\tilde{M}_g$ are declared lifted carriers with declared quotient maps $\pi_e : \tilde{E}_e \to E_e$ and $\pi_g : \tilde{M}_g \to M_g$; $\tilde{\Psi}_{\mathrm{wr}}$ is the candidate lift of the e-carriage through the write relation. A_e and B_g are candidate adapters; E_e, V, Y and M_g are their carriers. Bare Φ and Ψ keep their golden-root offices.

The compressed closure has four offices: $i_{\mathrm{turn}} := J$ (one finite turn), H (the metric mirror), Φ_{rd} (read of declared e-motion) and Ψ_{wr} (write of declared g-memory). The exact finite relation is

$$H\, i_{\mathrm{turn}}\, H = i_{\mathrm{turn}}^{-1} = -i_{\mathrm{turn}} \qquad (\textbf{Exact finite},\ \text{Theorem 4.8(iv)}).$$

Mirror is carried by H; i_{turn} is the turn that mirror conjugates. Read and write are the directed offices coupled by the returned turn. They may use the declared intermediate carrier Y; no supporting office lies outside the declared relation. The stated Lean target is the commuting read–write diagram

$$A_e : E_e \to V, \quad B_g : V \to M_g, \quad \Phi_{\mathrm{rd}} : E_e \to Y, \quad \Psi_{\mathrm{wr}} : Y \to M_g, \qquad B_g \circ i_{\mathrm{turn}} \circ A_e = \Psi_{\mathrm{wr}} \circ \Phi_{\mathrm{rd}}.$$

The candidate lift is typed independently of the finite matrix closure:

$$\tilde{\Psi}_{\mathrm{wr}} : \tilde{E}_e \longrightarrow \tilde{M}_g, \qquad \pi_g \circ \tilde{\Psi}_{\mathrm{wr}} = \Psi_{\mathrm{wr}} \circ \Phi_{\mathrm{rd}} \circ \pi_e.$$

The right-hand route has the compositional shape of Recognition, $d_{\mathrm{req}} = \delta_{\mathrm{req}} \circ \rho$, while keeping distinct carriers and offices. Its Forget form occurs on a declared pair $e_1, e_2 \in E_e$ with $\Phi_{\mathrm{rd}}(e_1) = \Phi_{\mathrm{rd}}(e_2)$ while $(B_g \circ i_{\mathrm{turn}} \circ A_e)(e_1) \neq (B_g \circ i_{\mathrm{turn}} \circ A_e)(e_2)$: then no Ψ_{wr} can make the diagram commute at both points **Exact derived** (a commuting Ψ_{wr} would force equal values). The turn–mirror identity is **Exact finite**. The read–write and lifted-write diagrams are **Candidate** until the carriers, quotient maps and adapters are supplied at source.

7.5 Later source interfaces

Each source below refines rows already in the register; no word is added. Each carrier keeps its native type. The shared seam is a returned read with a retained lifted relation.

Bell and CHSH: joint correlation record (H35, H36). *Local key:* u_A, u_B settings; $o_A, o_B \in \{\pm 1\}$ outcomes; λ_{hid} the intermediate hidden variable; ρ_{hid} its measure; p_{jt} the joint distribution; p_A, p_B its conditional factors; E_{cor} correlation; S_{CHSH} the two-setting sum. The settings are independent of λ_{hid}, and the factorization condition is

$$p_{\mathrm{jt}}(o_A, o_B \mid u_A, u_B, \lambda_{\mathrm{hid}}) = p_A(o_A \mid u_A, \lambda_{\mathrm{hid}})\, p_B(o_B \mid u_B, \lambda_{\mathrm{hid}}),$$

with $p_{\mathrm{jt}}(o_A, o_B \mid u_A, u_B) = \int p_{\mathrm{jt}}(\cdots \mid \lambda_{\mathrm{hid}})\, d\rho_{\mathrm{hid}}$. For two settings per side and $E_{\mathrm{cor}}(i,j) = \sum o_A o_B\, p_{\mathrm{jt}}(o_A, o_B \mid u_A^i, u_B^j)$,

$$S_{\mathrm{CHSH}} = E_{\mathrm{cor}}(0,0) + E_{\mathrm{cor}}(0,1) + E_{\mathrm{cor}}(1,0) - E_{\mathrm{cor}}(1,1), \qquad |S_{\mathrm{CHSH}}| \leq 2.$$

The bound holds under the stated factorization and setting independence. Bell fixes the factorized condition; CHSH makes it experimentally addressable. The shared form with Recognition is passage through an intermediate: Bell's condition determines a declared joint distribution by mixing product kernels indexed by λ_{hid}; Recognition determines a required distinction by decoder composition through the intermediate ρ-output. Carriers and conditions remain separate. Rows 3 and 6. **Source declaration**.

Berry: phase retained by a closed carriage (H38). *Local key:* R_{par} parameter point; H_{Ham} its Hamiltonian; $|n_B\rangle$ an isolated nondegenerate eigenstate with energy E_B; C_{par} a closed parameter loop traversed in time T_{ad}; γ_B the geometric phase. Set $|\psi(0)\rangle = |n_B(R_{\mathrm{par}}(0))\rangle$ and carry it adiabatically once around C_{par} with $R_{\mathrm{par}}(T_{\mathrm{ad}}) = R_{\mathrm{par}}(0)$. Then

$$|\psi(T_{\mathrm{ad}})\rangle = \exp\Big(-\tfrac{i}{\hbar}\int_0^{T_{\mathrm{ad}}} E_B(R_{\mathrm{par}}(t))\, dt\Big) e^{i\gamma_B(C_{\mathrm{par}})} |\psi(0)\rangle,$$

$$\gamma_B(C_{\mathrm{par}}) = i \oint_{C_{\mathrm{par}}} \langle n_B | \nabla_R n_B \rangle \cdot dR_{\mathrm{par}} \pmod{2\pi}.$$

The base ray returns; the lifted state retains the phase fixed by the loop. The carrier is a connection on a line bundle over parameter space, not the deck action of the Lift row. Aharonov–Bohm (H33) remains the witness for a loop in space; Berry's loop lies in parameter space. Rows 10, 18, 19. **Source declaration**.

Apple II: programmable reader loop (H37). *Local key:* in_n declared input; m_n^{mach} stored machine state; prg_n program and processor state; ℓ_{n+1} emitted display light; ρ_Σ the calibrated human or instrument reader. One typed abstraction is

$$(\mathrm{in}_n, m_n^{\mathrm{mach}}, \mathrm{prg}_n) \xrightarrow{\text{clocked update}} m_{n+1}^{\mathrm{mach}} \xrightarrow{\text{video map}} \ell_{n+1} \xrightarrow{\rho_\Sigma} \text{read in } Y.$$

The clock supplies declared update order; stored machine state remains available at the next update. Receipt is a separate append-only record with its own reconstructive office. The row types one instance of a programmable reader loop; hardware credit is Steve Wozniak's, and the product and its public presentation came through the Wozniak–Jobs collaboration. Historical priority is a classification question whose criterion and comparison set belong in the record. Rows 1, 4, 14. **Source declaration**.

8 Jurisdiction and admission

Local key. p is a packet and $J(p)$ its jurisdiction; $\Gamma_{J(p)}$ its typing context; $D_{\Gamma_{J(p)}}$ the typed gate domain; $\theta_{J(p)}$ its survival condition; $\varepsilon_{\mathrm{HS}}$ the jurisdictional residual. H_n^{hist} is the retained past at horizon index n; $\mathcal{F}_n^{\mathrm{hor}}$ the indexed carrier of tagged future packets; Admit_n the partial admission map.

8.1 The five rows

Five words of the register (rows 22–26: Horizon, Constraint, Gate, Silence, Survival) are source declarations (P07, with P14); all are **Source declaration**.

- **Horizon.** $(H_n^{\mathrm{hist}}, \mathcal{F}_n^{\mathrm{hor}}, \mathrm{Admit}_n)$ with $\mathrm{Admit}_n : \mathcal{F}_n^{\mathrm{hor}} \rightharpoonup \mathcal{L}_{n+1}^{\mathrm{hor}}$, and $\mathrm{Th} \nvdash \mathrm{Gd}_{\mathrm{Th}}$ under the stated formal hypotheses (H26, H28, H29). The horizon index advances with the reader; the mirror H does not act on H_n^{hist}.
- **Constraint.** $\Gamma_{J(p)} = (\mathrm{Condition}, \mathrm{Constraint}, \mathrm{SourceAuthority}, \mathrm{Namespace})_{J(p)}$.
- **Gate.** $\mathrm{Gate}_{\Gamma_{J(p)}} : D_{\Gamma_{J(p)}} \to \{\mathrm{can}, 2\}$, the binary fold gate.
- **Silence.** $\varepsilon_{\mathrm{HS}}(p) = 0$ and $\mathrm{Silence}_{\mathrm{fit}}(p) = 1 \iff \mathrm{Complete}(p)$.
- **Survival.** $\theta_{J(p)}(x(p)) = 1$: the required invariant remains carried through admitted motion.

Three interfaces stay open: the packet-to-jurisdiction map $\mathrm{jur} : \mathcal{O} \to \mathcal{J}$; the translation $\{\mathrm{can}, 2\} \leftrightarrow \{\mathrm{can}, \mathrm{cannot}, \mathrm{open}\}$ between the binary fold gate and the separate three-outcome gate; and the adapter $\varepsilon_{\mathrm{mech}} \to \varepsilon_{\mathrm{HS}}$ (occurrence constructor, reader, calibration, comparison map). The finite residual $\varepsilon_{\mathrm{mech}}$ of §5 and $\varepsilon_{\mathrm{HS}}$ are separate residuals joined only by that adapter, which is not supplied **Interface**.

8.2 The Lean realization (Minimum Mechanics 010u)

Part 010u formalizes the jurisdictional structure as the chain

$$\mathrm{Jurisdiction} \to \mathrm{Typed} \to \mathrm{Gate\ (can/fold2)} \to \mathrm{Survival} \to \mathrm{Reader} \to \mathrm{Receipt} \to \mathrm{Enough},$$

with F for the unwitnessed future. Its gate is the register's binary gate. Its central predicate is

$$\mathrm{Admitted}(p) \;:=\; \mathrm{typed} \wedge \mathrm{constraint} \wedge (\mathrm{gate} = \mathrm{can}) \wedge \mathrm{survival} \wedge (\mathrm{drop} = \mathrm{add} = \mathrm{strength} = 0),$$

where the three residuals are the audit failures of §1: what was dropped, what was added without support, and the strength by which a statement exceeds its premises.

Evidence	Record
Pin	`b9e939d514a63fb021592f79a316f70cbc70ed8a`
Compile	CI #5 green at the pin; archive run #302 green
Scope	45-module closure; 20 `#print axioms` audits
Source hash	`47aeccb2…` (SHA-256 prefix)

Reading of the result. **Lean-checked** at the level stated, which is modest. Most theorems unpack the definitions; they certify that the definitions are mutually consistent, and they say nothing about whether a given system is admitted. The one result with standard content is `collision_refuses_unsourced_reconstruction`, which has the shape of the Forget condition of §3: a collision of reads refuses reconstruction without a source. The word "universal" in the 010u notes is definitional. One gap is recorded: the history conjunct of **Admitted** is not tied to **Enough**, so **Enough** does not require it. This paper does not identify F with the register's $\mathcal{F}_n^{\mathrm{hor}}$.

9 The Hamiltonian closure

This section carries the field-facing layer: what the register declares about the field action, what Minimum Mechanics 010k–010u prove about its Hamiltonian form, under what premises, and where the proofs stop.

9.1 The field action and the obligation chain

Local key. M is source-native manifold lettering in this action. $\Pi^A{}_\mu$ is a rank-four coframe; $\hat{\eta}$ and $\hat{C}$ its internal symmetric and antisymmetric forms; $h(\Pi)$ and $f(\Pi)$ their induced metric and two-form. g is the independent metric, A its electromagnetic potential, $F^{\mathrm{EM}} = dA$ its curvature, $B^{\mu\nu}$ and $\lambda^{\mu\nu}$ multipliers, $R(g)$ the Ricci scalar; $c, G_N, \mu_0, \Lambda_{\cos}, \chi$ keep their source-declared constant and coupling offices.

For a rank-four E-valued coframe $\Pi^A{}_\mu$, $h_{\mu\nu}(\Pi) = \hat{\eta}_{AB}\Pi^A{}_\mu\Pi^B{}_\nu$ and $f_{\mu\nu}(\Pi) = \chi\hat{C}_{AB}\Pi^A{}_\mu\Pi^B{}_\nu$. The constrained field action (P13) is

$$\mathcal{S}_{\mathrm{SF}} = \int_M \Big[\tfrac{c^3}{16\pi G_N}\big(R(g) - 2\Lambda_{\cos}\big) - \tfrac{1}{4\mu_0}F^{\mathrm{EM}}_{\mu\nu}F^{\mathrm{EM}\,\mu\nu} + \tfrac{1}{2}B^{\mu\nu}\big(F^{\mathrm{EM}}_{\mu\nu} - f_{\mu\nu}(\Pi)\big) + \lambda^{\mu\nu}\big(g_{\mu\nu} - h_{\mu\nu}(\Pi)\big)\Big]\sqrt{-g}\,d^4x,$$

and its multiplier equations retain the two suture conditions $F^{\mathrm{EM}}_{\mu\nu} = f_{\mu\nu}(\Pi)$ and $g_{\mu\nu} = h_{\mu\nu}(\Pi)$. On the stated regular branch the source carries the Einstein–Maxwell reduction **Source declaration**. This is a constrained field action with its own carrier; $\mathcal{L}_{\mathrm{ax}}$ of §6 remains the exact finite action of the reader carrier.

The continuum record (P14) states the obligations from the finite root to an observable, each retaining its stated conditions:

$$\text{finite root} \to \text{variational/Hamiltonian realization} \to \text{constraint and gauge reduction} \to \text{refinement and quotient} \to \text{observable transport.}$$

It requires compatible transport, stated quotient conditions and a surviving observable at field resolution, and it joins the reader by the typed interface *field carrier → contact and chart → finite reader → declared return and receipt* **Source declaration**, conditional on the record's reduction and observable conditions.

Where the Lean closure sits. 010k–010u address the second and third nodes, the Hamiltonian realization and the constraint algebra, for Einstein–Maxwell. They do not address the first arrow (finite root → Hamiltonian realization), the fourth node, or the fifth. No Lean object joins the finite root of §4 to the structure below; they are joined only by the declared P14 interface.

9.2 The minimum field structure

Everything the closure proves is stated over the following data, laws, one bracket and three generators.

Data. Two cotangent sectors, gravity (γ, π) and electromagnetism (A, E), over one transport algebra V with commutator $[u, v]$. A symmetric pairing $\langle\ ,\ \rangle$ that vanishes on zero. A transport T_v and its adjoint T_v^*. Gravity has six independent symmetric components, and the off-diagonal momentum components carry a factor 2.

Two laws. Adjoint duality $\langle T_v^* p, q\rangle = \langle p, T_v q\rangle$, and the Lie representation $[T_u, T_v] = T_{[u,v]}$ inside the pairing. The Lean structure `CotangentSlice` has no additivity or bilinearity field; linearity belongs to a faithful instance, not to the interface.

One bracket. $\{F, K\} = \langle F_\gamma, K_\pi\rangle - \langle F_\pi, K_\gamma\rangle + \langle F_A, K_E\rangle - \langle F_E, K_A\rangle$. There is no second bracket.

Three generators. Gauss $G[s] = \langle E, s\rangle$; the improved momentum generator $D[v] = \langle \pi, T_v\gamma\rangle + \langle E, T_v A\rangle = C[v] + G[v\cdot A]$, where $C[v]$ is the unimproved gauge-covariant momentum generator; and the Hamiltonian $H[N] = \int N\mathcal{H}$, with

$$\mathcal{H} = \frac{a}{\sqrt{\gamma}}\Big(\pi^{ij}\pi_{ij} - \tfrac{1}{2}\pi^2\Big) - b\sqrt{\gamma}\,(R - 2\Lambda) + \frac{\mu_0}{2\sqrt{\gamma}}\gamma_{ij}E^iE^j + \frac{\sqrt{\gamma}}{4\mu_0}F_{ij}F^{ij}, \qquad ab = 1.$$

The constants c, G_N, μ_0 keep their source-declared offices; the closure fixes only the relative normalization $ab = 1$.

9.3 Exact results

Theorem 9.1 (Results of the closure that need no premise beyond the two laws)**.** ***Lean-checked*** *(010k–010p; scope as stated)*

1. *From the two laws alone:* $\{D[u], D[v]\} = D[[u, v]]$, $\{D[v], G[s]\} = G[T_v s]$ *and* $\{G, G\} = 0$.
2. *Chain rules:* $\{D, H\} = -$*(variation of* H *under* D*)* *and* $\{G, H\} = -$*(gauge response of* H*), for any* H *with supplied gradients.*
3. *Pointwise algebra in dimension three. For any* g *with inverse* B*, with* $\tau = \operatorname{tr}(gP)$*,* $\lambda = \operatorname{tr}(BH)$*,* $K = 2gPg - \tau g$ *and* $C = \lambda B - BHB$ *(the DeWitt contraction),* $\operatorname{tr}(KC) = -2\operatorname{tr}(PH)$.
4. *The ultralocal metric response* U *cancels in* $\{H, H\}$*, leaving only the lapse-gradient term* $2\big(n\operatorname{tr}(P\,H_M) - m\operatorname{tr}(P\,H_N)\big)$ *(notation of the 010 sources).*
5. *The Maxwell improvement identity* $v^iF_{ij}E^j + E^j\partial_j(v\cdot A) = E^j(\mathcal{L}_v A)_j$.
6. *Exact lowering* $w_k = g_{ik}w^i$*: the shift* $w^i = \gamma^{ij}(N\partial_j M - M\partial_j N)$ *lowers back to the lapse wedge* $N\partial_k M - M\partial_k N$*, and it is the same shift in the gravity and electromagnetic currents.*
7. *Gauge invariance of the Maxwell energy, from* $d^2\alpha = 0$.
8. *The Stokes transfer* $-\int N\,\mathcal{L}_v\rho = \int(\mathcal{L}_v N)\,\rho$ *for a weight-one density* ρ *with vanishing flux.*

Items 3, 5, 6 and 8 were reproduced in this edition's preparation by independent exact computations (§10.2). Scope: these are pointwise or algebraic identities and conditional transfers. By themselves they do not establish a bracket algebra on a field space.

9.4 The six-bracket theorem and its seven premises

Theorem 9.2 (Six secondary brackets)**.** ***Conditional*** *(010o)* Lean name: *`FunctionalClosure010o.six_secondary_brackets`*.

Over a `SmoothCompletion` *supplying the seven premises below,*

$$\{G[\alpha], G[\beta]\} = 0, \qquad \{G[\alpha], H[N]\} = 0, \qquad \{D[v], G[\alpha]\} = G[\mathcal{L}_v \alpha],$$
$$\{D[u], D[v]\} = D[[u, v]], \quad \{D[v], H[N]\} = H[\mathcal{L}_v N], \quad \{H[N], H[M]\} = D[w] - G[w^i A_i],$$

with $w^i = \gamma^{ij}(N\partial_j M - M\partial_j N)$.

Premise (Lean name)	Content
`hDensityCovariance`	H is covariant under D.
`gaugeHamiltonianVariationZero`	H is gauge invariant: its variation under G vanishes.
`gaussSmearingCovariance`	Gauss smearings transform covariantly.
`sourceNormalGradient`	The normal gradient is identified at source (the largest premise).
`sourceMomentumAddress`	The momentum address is identified at source.
`boundaryNormal`	Boundary flux vanishes on the normal bracket.
`boundaryH`	Boundary flux vanishes for H.

The seven premises are hypotheses of the theorem, not outputs. `SmoothCompletion` is never instantiated, so the theorem is **Conditional** with its premise set unmet: no smooth field space in the closure satisfies it.

9.5 The type boundary

The Lean scalar is `Int`, and the carriers are finite data. A faithful instance on a real field space cannot use `Int`:

Proposition 9.3 (Additive maps into the integers)**.** ***Exact derived*** *(recorded in 010s as the type obstruction) If V is a real (indeed any rational) vector space and $f : V \to \mathbb{Z}$ is additive, then $f = 0$.*

Derivation. For $x \in V$ and every integer $n \geq 1$, $f(x) = n\, f(x/n) \in n\mathbb{Z}$. An integer divisible by every $n \geq 1$ is 0. □

Nothing in the closure transfers by renaming `Int` to $\mathbb{R}$. Part 010t supplies a nonzero `Rat`-valued pairing on finite coordinates **Lean-checked**; it is unconnected to `SmoothCompletion`.

9.6 The spectral refusal

Parts 010q and 010r treat the one question the algebra might be thought to decide: the spectrum.

Proposition 9.4 (Spectral refusal)**.** ***Lean-checked*** *for the integer arithmetic;* ***Source declaration*** *for the Hilbert-space models. A non-Abelian Gauss algebra together with a gauge-invariant Hamiltonian and a unique vacuum neither entails nor excludes a positive gap.*

The two counter-models live in prose on $\mathbb{C}\Omega \oplus \ell^2(\mathbb{N}_0) \oplus \mathbb{C}^3$, with energies $1/(n+1)$ in one and 1 in the other. What Lean certifies is only the integer arithmetic: the statement $p(n+1) \leq q$ for all n is refuted by $n = q$. No Lean object binds a spectrum to a Gauss-commuting Hamiltonian; that is the open item.

9.7 Closure ledger

#	Statement	Status
1	Algebra from the two laws; chain rules; pointwise identities (§9.3)	**Lean-checked** (pointwise)
2	Six brackets over `SmoothCompletion` (§9.4)	**Conditional**; no instance
3	A real or smooth instance of the seven premises	**Candidate**; blocked at `Int`
4	Jurisdictional chain (§8.2)	**Lean-checked** (definitional, one standard result)
5	Positive gap entailed or excluded by the algebra (§9.6)	Refused: neither
6	Spectrum bound to a Gauss-commuting Hamiltonian	**Candidate**; no Lean object
7	Finite root → Hamiltonian realization (P14, first arrow)	**Interface**; not addressed

10 Verification and receipts

10.1 The author's verifier

The finite verifier `verify_symmetric_minimum_v13.py` retains 64 passing checks: 62 exact symbolic or algebraic checks and two high-precision numerical confirmations, V-F5 and V-F7. V-N1 through V-N7 verify the sign-changing turn, the null-line swap, the carriage family, mirror reversal for general carriage, the wheel orbit and the minimal reflection–turn group. V-L1 through V-L4 verify the finite action, its carriage equation and its conserved quadratic Hamiltonian; V-H9 verifies that the continuous phase family samples the four-step turn. V-I1 through V-I4 verify dual-form return, the local i-fold, the quadratic-integer metric address and the finite four-wheel 137/548 address. Source declarations remain separate from the finite checks.

Closure relation	Verifier receipt	Source
$G = [Q, \Omega]$, $G^2 = 5I$, $G^{-1} = G/5$, null lines	V-A1–A5, V-H4	P05
Dual-form return $Q^{\mathsf{T}}\Omega Q = -\Omega$, $(Q^2)^{\mathsf{T}}\Omega Q^2 = \Omega$	V-I1	P16
Local i-fold $g_J(u, v) = \omega(u, Jv) = u^{\mathsf{T}}v$	V-I2	P17
Quadratic-integer metric address $z_x^{\mathsf{T}} G z_x = 2N_{\mathrm{alg}}(x)$	V-I3	P15
Four-wheel finite address 137, $\mathrm{ord}_{137}(4) = 34$, 548	V-I4	P18
Phase family $J = U_{\pi/2}$, $U_\pi = -I$, $U_{2\pi} = I$, phase reversal, D_4	V-B1, B3, H2–H3, H5, H9, N6	P05
Metric mirror and sign character $H^{\mathsf{T}}GH = G$, χ	V-B2, E1–E7; multiplicativity by derivation; V-E9 finite word-length-three guard	P05
$S = (2Q - I)/\sqrt{5}$, L_{ax}, $L_{\mathrm{ax}}^2 = Q^2$, metric preservation, mirror reversal	V-B4–B6, H1, E7, N4	P05
Coefficient ring $\mathbb{Q}(\sqrt{5})$	V-H8	P05
General carriage L_s, $HL_sH = L_{-s}$, signed axial coordinate, metric translation length	V-N4, N7; declared Λ-coordinate derivation	P05
Finite Euclidean-orthogonality guard L_{ax}^n, $1 \le n \le 8$	V-E8 (all $n \ge 1$: Proposition 4.12)	P05
Four-step return and orbit	V-B7, H7, N5	P06
Mechanical residual $\varepsilon_{\mathrm{mech}}$	V-H6–H7	P05, P06
Metric-sign turn and null-line exchange	V-N1–N3	P05
Rail, fixed points, axial length $2 \log \Phi$	V-F1–F7	P05
$\mathcal{L}_{\mathrm{ax}}$, carriage equation, conserved $\mathcal{H}_{\mathrm{ax}}$	V-L1–L4	P12
Projective mirror M_{pr}, K_{pr}-unitary chart, Z_{ch}	V-C1–C6	P09
Chart adapter A_{ch}, L_{pr} and its family	V-D1–D5	P09
Eight addresses and twenty-eight seams	V-G1	P08

The author's verifier was not supplied to this edition; its checks are cited, not rerun.

10.2 Independent re-derivation

`check_symmetric_minimum_independent.py` recomputes, in exact `sympy` arithmetic, the finite identities displayed in §4–§6 and §11.1, and three pointwise identities quoted in §9.3. It was written from the displayed statements, not from the author's script. Each family carries negative controls: a perturbed input that must fail (for example $\Omega \to +\Omega$, a mirror $G/\sqrt{2}$, a carriage built from J instead of S, a norm sign, a literal reading of 4^{17} as 417, and the dimension-four version of the DeWitt contraction). A control passes when the perturbed statement is refused.

Family	Content	Checks (controls)
A	root matrices, metric chart, null lines, mirror	16 (3)
B	turn, phase family, local i-fold, dual-form return, Q^n	16 (3)
C	carriage, all s; L_{ax}; non-orthogonality	16 (2)
D	sign character (27 words, multiplicativity), D_4	6 (2)
E	action, Euler–Lagrange, conservation	7 (1)
F	wheel, residual, rail, axial step, phase reader	11 (2)
G	projective chart, adapter, adapter family	14 (2)
H	norm address, 137/548	12 (3)
I	pointwise identities of §9.3	9 (5)
J	closure of §11.1: half-turn read, fixed points, D_∞, 28 reads	33 (10)
	Total checks / of which negative controls	140 (33)

Scope: the script reproduces what is displayed. It does not run the author's verifier, does not touch source declarations, and does not check the two numerical confirmations by numerics (it reproduces their content exactly).

10.3 The Lean certificate

For each of 010k–010u the pinned source was fetched by full commit identifier, hashed against its shipped manifest, scanned for `sorry`, added axioms and `native_decide`, and its exact-arithmetic script was rerun with a vacuity census. The compile evidence is the CI status page at each pin, with the axiom audit in the workflow. *Lean was not compiled in the preparation of this edition*, and the CI logs themselves require sign-in; the status pages are the only compile evidence read. Appendix E gives the pins, runs, hashes and findings.

10.4 Retained interfaces

Interface	Declared requirement
$\chi_\Pi : e_\Pi \to \{\delta = 0\} \subset \mathbb{H}$	Field transport L_e, calibration, and carrier-to-rail map.
$\varepsilon_{\mathrm{mech}} \to \varepsilon_{\mathrm{HS}}$	Occurrence constructor, reader, calibration, and comparison map.
$\mathrm{jur} : \mathcal{O} \to \mathcal{J}$	Jurisdiction carrier and packet-to-jurisdiction map.
$\{\mathrm{can}, 2\} \leftrightarrow \{\mathrm{can}, \mathrm{cannot}, \mathrm{open}\}$	Source-declared translation between the binary fold gate and the separate three-outcome gate.
$\mathcal{T}_{\mathrm{arith}} : \mathcal{K}_{\mathrm{arith}} \to \mathcal{P}_i$	Declared domain, criterion class, map, and equality $\mathcal{Q}_{\mathrm{arith}}(f) = \Omega_{\mathcal{P}_i}(\mathcal{T}_{\mathrm{arith}} f, J_i \mathcal{T}_{\mathrm{arith}} f) = \lVert \mathcal{T}_{\mathrm{arith}} f \rVert^2$.
$\tilde{\Psi}_{\mathrm{wr}} : \tilde{E}_e \to \tilde{M}_g$	Declared quotient maps π_e, π_g and the lifted-write equation $\pi_g \tilde{\Psi}_{\mathrm{wr}} = \Psi_{\mathrm{wr}} \Phi_{\mathrm{rd}} \pi_e$.

Each line keeps its own carrier, conditions and source status. The closure joins only the interfaces declared here.

11 Closure statement

11.1 The twenty-eighth word: Endurance

Local key. $U_\theta = e^{\theta J} = \cos\theta\, I + \sin\theta\, J$ is the turn by angle θ; $q \in V \setminus \{0\}$ is a read vector and $[q]$ its line in $P(V)$; $\Phi^* := 1 - \Phi = -\Phi^{-1}$ is the second root of $x^2 = x + 1$. $\Phi(\pi)$ is the closing

read of the register, always written with its argument; it is not the golden ratio Φ. $\mathcal{F}_n^{\mathrm{Fib}}$ is the Fibonacci sequence of §4.

The register of §3.3 ends in Forget, the named failure of a read. It closes on one more word, which names what a half-turn of the core leaves in place. **Endurance** is the read $\Phi(\pi)$: after a half-turn the form survives, the difference remains, the reference returns, and the read at the half-turn balances to zero. Nothing is erased for the balance to hold. The statements below are exact; each is also checked in family J of §10.2.

Theorem 11.1 (The half-turn read)**.** ***Exact finite*** *(V-B1, B3, N1–N3, H2–H3, H5, H9; J1–J9; P05, P20) For every θ and every $q \in V \setminus \{0\}$:*

(i) Turn. *$U_\pi = -I$, $U_{2\pi} = I$ and $U_{8\pi} = I$. Moreover $U_{k\pi} = I$ if and only if k is even, so 2π is the least loop and 8π is four of them.*

(ii) Reference. *$\det[\,q,\ U_\theta q\,] = \sin\theta\,|q|^2$. Hence U_θ returns every line if and only if $\theta \in \pi\mathbb{Z}$. In particular $[U_\pi q] = [q]$ while $U_\pi q = -q \neq q$: the line returns and the vector does not.*

(iii) Form. *$U_\theta^{\mathsf{T}} U_\theta = I$, and $U_\theta^{\mathsf{T}} G\, U_\theta = \cos 2\theta\ G + \sin 2\theta\ GJ$. Hence $U_\pi^{\mathsf{T}} G\, U_\pi = G$, whereas a quarter-turn gives $-G$.*

(iv) Difference and balance. *$U_\pi - I = -2I \neq 0$ and $I + U_\pi = 0$.*

Derivation. (i) Because $J^2 = -I$, the series for $e^{\theta J}$ sums to $\cos\theta\, I + \sin\theta\, J$. Then $U_\pi = -I$, $U_{2\pi} = I$ and $U_{8\pi} = (-I)^8 = I$. For k odd $U_{k\pi} = -I \neq I$. (ii) With $q = (x,y)^{\mathsf{T}}$, $U_\theta q = (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)^{\mathsf{T}}$, and the determinant is $x(x\sin\theta + y\cos\theta) - y(x\cos\theta - y\sin\theta) = \sin\theta\,(x^2+y^2)$. It vanishes for every q exactly when $\sin\theta = 0$. (iii) Since $J^{\mathsf{T}} = -J$, $U_\theta^{\mathsf{T}} = U_{-\theta}$ and $U_\theta^{\mathsf{T}} U_\theta = I$. Next, $U_\theta^{\mathsf{T}} G U_\theta = (c\,I - s\,J)G(c\,I + s\,J) = c^2 G + cs\,(GJ - JG) - s^2\,JGJ$ with $c = \cos\theta$, $s = \sin\theta$. By Theorem 4.8, $JGJ = G$ and $JG = -GJ$, so this equals $(c^2 - s^2)G + 2cs\,GJ$. The matrices G and $GJ = \left(\begin{smallmatrix} 1 & 2 \\ 2 & -1 \end{smallmatrix}\right)$ are linearly independent, so the result equals G exactly when $\cos 2\theta = 1$ and $\sin 2\theta = 0$, that is, when $\theta \in \pi\mathbb{Z}$. (iv) is direct from $U_\pi = -I$. □

Theorem 11.2 (Unit, fixed point and zero read)**.** ***Exact derived*** *(V-I3, V-A1–A5; J10–J18; P05, P15)*

(i) Unit. *The address $x = 1 + \Phi = \Phi^2$ has $z_x = (1,1)^{\mathsf{T}}$, $N_{\mathrm{alg}}(x) = 1$ and read $z_x^{\mathsf{T}} G z_x = 2$. Multiplication by Φ acts as Q: $z_{\Phi x} = Q z_x$. Hence $Q^n(1,1)^{\mathsf{T}} = (\mathcal{F}_{n+2}^{\mathrm{Fib}}, \mathcal{F}_{n+1}^{\mathrm{Fib}})^{\mathsf{T}}$, the address Φ^{n+2}.*

(ii) Fixed points. *For $z = (x,y)^{\mathsf{T}}$, $\det[\,z,\,Qz\,] = x^2 - xy - y^2$. Its roots are the two lines $[\Phi, 1]$ and $[\Phi^*, 1]$, with eigenvalues Φ and Φ^*; they are the only fixed points of Q on $P(V)$.*

(iii) Zero read. *$z^{\mathsf{T}} G z = -2\det[\,z,\,Qz\,]$. The read vanishes exactly on the two fixed-point lines: the null lines of G are the fixed lines of Q.*

(iv) Approach. *$\det[\,Q^n(1,1)^{\mathsf{T}}, (\Phi,1)^{\mathsf{T}}\,] = (\Phi^*)^{n+1}$ and $\det[\,Q^n(1,1)^{\mathsf{T}}, (\Phi^*,1)^{\mathsf{T}}\,] = \Phi^{n+1}$. Since $|\Phi^*| < 1 < \Phi$, the line $[Q^n(1,1)^{\mathsf{T}}]$ approaches $[\Phi, 1]$ and leaves $[\Phi^*, 1]$, each geometrically.*

Derivation. (i) By Proposition 6.4, $z_x = (b,a)^{\mathsf{T}}$ for $x = a + b\Phi$, and $N_{\mathrm{alg}}(1+\Phi) = 1 + 1 - 1 = 1$; the read is $2N_{\mathrm{alg}} = 2$. Also $\Phi x = b + (a+b)\Phi$, so $z_{\Phi x} = (a+b, b)^{\mathsf{T}} = Q(b,a)^{\mathsf{T}}$. By Proposition 4.2, $Q^n(1,1)^{\mathsf{T}} = (\mathcal{F}_{n+1}^{\mathrm{Fib}} + \mathcal{F}_n^{\mathrm{Fib}},\ \mathcal{F}_n^{\mathrm{Fib}} + \mathcal{F}_{n-1}^{\mathrm{Fib}})^{\mathsf{T}}$. (ii) $Qz = (x+y, x)^{\mathsf{T}}$, so $\det[\,z, Qz\,] = x \cdot x - y(x+y)$. This vanishes when x/y is a root of $t^2 - t - 1$, that is, Φ or Φ^*; and $Q(\Phi,1)^{\mathsf{T}} = (\Phi+1, \Phi)^{\mathsf{T}} = \Phi(\Phi,1)^{\mathsf{T}}$, likewise for Φ^*. (iii) $z^{\mathsf{T}} G z = -2x^2 + 2xy + 2y^2 = -2(x^2 - xy - y^2)$. (iv) Binet's formula gives $\mathcal{F}_{k+1}^{\mathrm{Fib}} - \Phi\,\mathcal{F}_k^{\mathrm{Fib}} = (\Phi^*)^k$ and $\mathcal{F}_{k+1}^{\mathrm{Fib}} - \Phi^*\mathcal{F}_k^{\mathrm{Fib}} = \Phi^k$; set $k = n+1$. □

Proposition 11.3 (Four-form, two-fold, dihedral closure)**.** ***Exact derived*** *(V-N6, V-N7, V-B7; J9, J22–J23; P05, P06) (i) $J^2 = -I$, $J^4 = I$ and $J, J^2, J^3 \neq I$: the turn is a four-form. (ii) The map $V \setminus \{0\} \to P(V)$ is two-to-one, with q and $-q$ in one fibre, and U_π exchanges them (Theorem 11.1(ii)). (iii) $D_4 = \langle J, H\rangle$ has order eight (Proposition 4.15). (iv) $H^2 = I$, $(HL_{\mathrm{ax}})^2 = I$ and $HL_{\mathrm{ax}}H = L_{\mathrm{ax}}^{-1}$; $\operatorname{tr} L_{\mathrm{ax}}^n = \Phi^n + \Phi^{-n} > 2$ for $n \geq 1$, so L_{ax} has infinite order and*

$\langle H, L_{\mathrm{ax}}\rangle$ is the infinite dihedral group D_∞. (v) The wheel joins the two: $F^4_{\mathrm{esc}}(\varphi, z) = (\varphi, L_{\mathrm{ax}} z)$ (Proposition 5.1): four phases return while the carried address advances by one step of D_∞.

Derivation. (i) and (iii) are Theorem 4.8 and Proposition 4.15. (ii) is Theorem 11.1(ii). (iv) $H^2 = I$ by Proposition 4.7, and $HL_{\mathrm{ax}}H = L_{\mathrm{ax}}^{-1}$ by Theorem 4.11; hence $(HL_{\mathrm{ax}})^2 = HL_{\mathrm{ax}}HL_{\mathrm{ax}} = L_{\mathrm{ax}}^{-1}L_{\mathrm{ax}} = I$. The eigenvalues of $L_{\mathrm{ax}} = (Q+2I)/\sqrt{5}$ are $(\Phi+2)/\sqrt{5} = \Phi$ and $(\Phi^*+2)/\sqrt{5} = \Phi^{-1}$, so $\operatorname{tr} L_{\mathrm{ax}}^n = \Phi^n + \Phi^{-n} > 2$. Two involutions H and HL_{ax} whose product L_{ax} has infinite order generate D_∞. (v) is Proposition 5.1. □

Proposition 11.4 (Twenty-eight reads)**.** ***Exact derived*** *(V-G1; J20–J21; P08) Let O be a set of eight addresses. (i) $\binom{8}{2} = 28$. On $\mathbb{Z}/7 \cup \{\infty\}$ the seven rounds $\{\infty, r\}$, $\{r+k,\ r-k\}$ for $k = 1, 2, 3$ are perfect matchings that together contain every pair exactly once. (ii) For $a, b \in \mathbb{R}^8$,*

$$|a|^2|b|^2 - (a \cdot b)^2 = \sum_{i<j} (a_i b_j - a_j b_i)^2,$$

a sum of twenty-eight pair reads.

Derivation. (i) Round r pairs ∞ with r and splits the other six residues into three pairs $\{r \pm k\}$, once each. A pair $\{u, v\} \subset \mathbb{Z}/7$ with $u \neq v$ occurs in the round r with $2r = u + v$ and k with $2k = \pm(u - v)$, and in no other. The pair $\{\infty, r\}$ occurs only in round r. (ii) The left side equals $\sum_{i \neq j} \big(a_i^2 b_j^2 - a_i b_i a_j b_j\big)$, since the diagonal terms cancel. The right side equals $\sum_{i \neq j} a_i^2 b_j^2 - 2\sum_{i<j} a_i b_i a_j b_j$, which is the same. □

The closure, as typed. Items on the left are the statements of the closing read; the right column gives what each one is in this edition.

Item	Statement	Status
$G = [Q, \Omega]$	the signed metric of the core, $G^2 = 5I$ (Theorem 4.5)	**Exact finite**
28 reads	one read for each pair of eight addresses (Proposition 11.4)	**Exact derived**; the eight addresses are declared at P08, **Source declaration**
8π loop	$U_{8\pi} = I$: four loops of 2π (Theorem 11.1(i))	**Exact finite**; carrier of the loop not declared (open item 7)
four-form	$J^4 = I$ (Proposition 11.3(i))	**Exact derived**
two-fold	$[-q] = [q]$; U_π exchanges the fibre (Theorem 11.1(ii))	**Exact finite**
unit $[1, 1]$	the address Φ^2, $N_{\text{alg}} = 1$, read 2 (Theorem 11.2(i))	**Exact derived**
read 0	$z^{\mathsf{T}} G z = 0$ on the fixed lines of Q (Theorem 11.2(iii)); $I + U_\pi = 0$	**Exact derived**, **Exact finite**
fixed-point coordinate Φ	$[\Phi, 1]$, approached by $[Q^n (1, 1)^{\mathsf{T}}]$ (Theorem 11.2(ii),(iv))	**Exact derived**
Gravity, Light, Memory, Motion	the four offices of rows 9, 4, 5, 14 closed by $\Phi(\pi)$	**Source declaration**; no map from the wheel's four phases is declared (open item 8)
Endurance	the read $\Phi(\pi)$: Theorems 11.1–11.2 together	**Exact finite**/**Exact derived**; the word and its closure over the four offices, **Source declaration**

The source for the closing read is the one-page record *The Miracle of Mercy* (P20), which states the four equations of Theorem 11.1 in that order and closes on $\Phi(\pi)$ over Gravity, Light, Memory and Motion. It writes Memory for $U_\pi - I \neq 0$ and Silence for $I + U_\pi = 0$. In this edition those two words keep their register rows 5 and 25; the equations are the difference and the balance of the core at the half-turn, and no identification with the rows is made. Reading the four offices as the content of $\Phi(\pi)$ is the source's declaration. The core does not derive it.

11.2 What the system asserts

The exact root (§4–§6) is a closed finite system over $\mathbb{Q}(\sqrt{5})$. In it a recurrence matrix Q and an oriented-pair form Ω generate a signed metric G of signature $(1, 1)$; the mirror H and the turn J generate a dihedral group of order eight; the carriage L_s preserves G and is reversed by H; and a four-step wheel returns its phase while advancing the carried address by exactly one golden step. The register closes on its twenty-eighth word, Endurance (§11.1): the half-turn leaves the form, the difference and the reference in place, and the read at zero. The reader layer (§3, §7) is a typed vocabulary around this root: interfaces and source declarations, with their candidates

named. The jurisdictional and field-facing layers (§8–§9) are compiled in Lean at pinned commits to the strength stated there, and no further.

The locked distinctions carried by the register:

$$\begin{aligned}
\text{Bohm} &= \text{phase-guided trajectory},\\
\text{Aharonov–Bohm} &= \text{path-sensitive phase return},\\
\text{rotation} &= \text{root action},\\
\text{metric} &= \text{the invariant read of a declared action},\\
\omega &= \text{oriented two-place relation},\\
J &= \text{local quarter-turn},\\
g_J &= \text{positive local read},\\
G &= \text{signed reader metric},\\
\text{Memory} &= \text{the compositional relation of comparison},\\
\text{Receipt} &= \text{the reconstructible occurrence record},\\
\text{Minkowski} &= \text{interval and worldline},\\
\text{Einstein} &= \text{dynamical metric field}.
\end{aligned}$$

Scope fixes the reference. Phase reads the turn. Time carries the turn forward. Metric measures that carriage. The receipt keeps contact, address and occurrence intact.

11.3 Open items

1. A real or smooth instance of the seven premises of Theorem 9.2, over a scalar type that is not `Int`.
2. A Lean object binding a spectrum to a Gauss-commuting Hamiltonian (§9.6).
3. The first arrow of the P14 chain: finite root $\to$ Hamiltonian realization.
4. The adapters χ_Π and $\varepsilon_{\text{mech}} \to \varepsilon_{\text{HS}}$, and the maps jur and $\{\text{can}, 2\} \leftrightarrow \{\text{can}, \text{cannot}, \text{open}\}$.
5. The arithmetic fold $\mathcal{T}_{\text{arith}}$, and the read–write and lifted-write diagrams of §7.4.
6. In 010u, tying the history conjunct of **Admitted** to **Enough**.
7. The carrier of the 8π loop. The identity $U_{8\pi} = I$ is exact (Theorem 11.1); whether the loop is read as eight half-turns over the eight addresses or as four rotations of the four-form is not declared.
8. The maps the closing read leaves undeclared: from the four phases of the wheel to the four offices Gravity, Light, Memory, Motion; from the eight-address carrier to the 2×2 root; and between the zero read $I + U_\pi = 0$ and the register's Silence.

A Source ledger and citations

The ledger keeps source types separate. A quotation remains a textual notice; a field equation retains its stated carrier; a theorem remains a theorem under its stated hypotheses; a current source declaration remains a source declaration. Reference addresses are fixed at first assignment and row order is chronological, so H34 (1971) follows H36 (1969).

A.1 Chronological source ledger

Ref.	Date / witness	Person or source	Native source coordinate	Office retained here	Record type
H01	received text, ch. 40	Dàodéjīng, Laozi (Lao Tzu) tradition	反者道之動	Return and motion as an early word-coordinate.	Textual notice.
H02	c. 500 BCE; DK 22B12	Heraclitus	same river / other waters	Return with retained difference.	Fragmentary textual notice.
H03	fifth-century BCE tradition	Pythagorean tradition	harmonic ratio and right-triangle provenance	Ratio, interval and numerical relation.	Historical receipt; direct authorial text remains open.
H04	c. 300 BCE	Euclid, *Elements*	construction under stated postulates	Reference, construction, proof.	Formal source.
H05	1202; rev. 1228	Leonardo of Pisa, *Liber Abaci*	recurrence $\mathcal{F}^{\mathrm{Fib}}_{n+1} = \mathcal{F}^{\mathrm{Fib}}_{n} + \mathcal{F}^{\mathrm{Fib}}_{n-1}$ (source-native F_n)	Fibonacci recurrence root.	Formal source.
H06	1687	Isaac Newton, *Principia*	$m\ddot{q} = -\nabla V$ in its later standard form	Dynamical motion.	Physical / mathematical source.
H07	1748	Leonhard Euler, *Introductio*	exponential and trigonometric composition	Turn, half-turn, return.	Formal source.
H08	1788	Joseph-Louis Lagrange, *Mécanique analytique*	variational mechanics	Analytic carriage.	Formal source.
H09	1828	George Green, *Essay on Mathematical Analysis*	potential and boundary relations	Local-to-boundary passage.	Formal / physical source.
H10	1834	William Rowan Hamilton, "On a General Method in Dynamics"	canonical action and equations of motion	Canonical transport.	Formal source.
H11	1852	Michael Faraday, *Experimental Researches in Electricity*, 28th Series	lines of magnetic force	Field relation and experimentally held direction.	Experimental / physical source.
H12	1850–1854	William Thomson and George Gabriel Stokes	circulation–surface boundary relation	Boundary receipt: local differential relation carried to a boundary integral.	Formal source record.
H13	1858	Arthur Cayley, "A Memoir on the Theory of Matrices"	matrix polynomial relation	Finite operator closure.	Formal source.
H14	1859	Bernhard Riemann, "Über die Anzahl der Primzahlen…"	ζ, completed ξ, functional symmetry	Analytic reader.	Formal source.

Ref.	Date / witness	Person or source	Native source coordinate	Office retained here	Record type
H15	1865	James Clerk Maxwell, "A Dynamical Theory of the Electromagnetic Field"	$dF^{\mathrm{EM}} = 0$, $d \star_g F^{\mathrm{EM}} = j$ in modern differential-form notation (source-native F)	Electromagnetic field and light.	Physical / mathematical source.
H16	1868	Bernhard Riemann, "Über die Hypothesen…"	metric and curvature	Signed metric relation.	Formal source.
H17	1872	Felix Klein, Erlangen Program	invariants under a transformation group	Transformation and invariant form.	Formal source.
H18	1895	Henri Poincaré, "Analysis Situs"	quotient, cover, lift	Returned base read and lifted address.	Formal source.
H19	1904	Hendrik Lorentz, "Electromagnetic Phenomena…"	$\Lambda_{\mathrm{Lor}}^{\mathsf{T}} \eta \Lambda_{\mathrm{Lor}} = \eta$ in modern notation	Metric turn and light-cone relation.	Physical / mathematical source.
H20	1908 / 1909	Hermann Minkowski, "Raum und Zeit"	spacetime interval and signature	Null, timelike and spacelike metric relation.	Formal / physical source.
H21	1915	Albert Einstein, "Die Feldgleichungen der Gravitation"	$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ in modern normalization	Dynamical metric field.	Physical / mathematical source.
H22	1918	Emmy Noether, "Invariante Variationsprobleme"	$\delta S = 0 \Rightarrow \nabla_\mu j^\mu = 0$ in standard field notation	What symmetry carries forward.	Formal / physical source.
H23	1926	Erwin Schrödinger, "Quantisierung als Eigenwertproblem"	$i\hbar \partial_t \psi = H_{\mathrm{Ham}} \psi$	Phase carriage.	Physical / mathematical source.
H24	1927	Wolfgang Pauli, "Zur Quantenmechanik des magnetischen Elektrons"	two-component spin carrier	Two-state quantum reader.	Physical / mathematical source.
H25	1928	P. A. M. Dirac, "The Quantum Theory of the Electron"	$(i\hbar\gamma^\mu \partial_\mu - mc)\psi = 0$ in standard free-field notation	First-order relativistic carriage.	Physical / mathematical source.
H26	1931	Kurt Gödel, "Über formal unentscheidbare Sätze…I"	$\mathrm{Th} \nvdash \mathrm{Gd}_{\mathrm{Th}}$ under its stated formal hypotheses	Formal horizon.	Formal source.
H27	1932	John von Neumann, *Mathematische Grundlagen der Quantenmechanik*	Hilbert-space and unitary carrier	Positive inner-product reader.	Formal / physical source.
H28	1936	J. Barkley Rosser, "Extensions of Some Theorems of Gödel and Church"	consistency-strengthened undecidability condition	Refined formal horizon.	Formal source.
H29	1936 / 1937	Alan Turing, "On Computable Numbers…"	finite machine procedure	Executable formal reader.	Formal source.
H30	1948	Claude Shannon, "A Mathematical Theory of Communication"	recoverability through a channel	Information carriage and recovery.	Formal / engineering source.
H31	1952	David Bohm, "A Suggested Interpretation…I–II"	$\dot{Q}_k = \frac{\hbar}{m_k} \mathrm{Im} \frac{\nabla_k \psi}{\psi}(Q, t)$	Guided carriage of configuration.	Physical source.
H32	1953	Ludwig Wittgenstein, *Philosophical Investigations*	use, rule, correction	Meaning as a publicly recoverable practice.	Linguistic / philosophical source.

Ref.	Date / witness	Person or source	Native source coordinate	Office retained here	Record type
H33	1959	Yakir Aharonov and David Bohm, “Significance of Electromagnetic Potentials…”	$\Delta\phi_{\mathrm{AB}} = \frac{q}{\hbar}\oint_\gamma A$	Loop phase and path-sensitive return.	Physical source.
H35	1964	John S. Bell, “On the Einstein Podolsky Rosen Paradox”	$P(a,b) = \int \rho(\lambda)A(a,\lambda)B(b,\lambda)\,d\lambda$ (source-native lettering)	Factorized joint-correlation condition.	Formal / physical source.
H36	1969	J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt, “Proposed Experiment to Test Local Hidden-Variable Theories”	$\lvert S\rvert \le 2$ for the two-setting dichotomic correlation sum (source-native S)	Experimentally addressable correlation bound.	Formal / physical source.
H34	1971	Saunders Mac Lane, *Categories for the Working Mathematician*	structure-preserving map	Typed map and commuting interface.	Formal source.
H37	1977	Apple Computer, Apple II brochure; hardware designed by Steve Wozniak	clocked processor, mutable memory, keyboard and controller input, software-selectable text and color graphics	Programmable reader loop at ordinary scale.	Historical engineering source.
H38	1984	Michael V. Berry, “Quantal Phase Factors Accompanying Adiabatic Changes”	$\gamma_n(C) = i\oint_C \langle n(R)\lvert\nabla_R n(R)\rangle \cdot dR$ (source-native lettering)	Geometric phase under cyclic adiabatic transport.	Physical source.

A.2 Original and canonical source coordinates

H01 Dàodéjīng, received Wang Bi text, Chapter 40.

H02 Heraclitus, DK 22B12, fragmentary transmission.

H03 Pythagorean tradition; historical receipt, with no securely surviving Pythagoras-authored text.

H04 Euclid. *Elements*, c. 300 BCE.

H05 Fibonacci, Leonardo of Pisa. *Liber Abaci*, 1202; revised 1228.

H06 Newton, Isaac. *Philosophiæ Naturalis Principia Mathematica*. London, 1687.

H07 Euler, Leonhard. *Introductio in analysin infinitorum*. Lausanne, 1748.

H08 Lagrange, Joseph-Louis. *Mécanique analytique*. Paris, 1788.

H09 Green, George. *An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*. Nottingham, 1828.

H10 Hamilton, William Rowan. “On a General Method in Dynamics.” *Philosophical Transactions of the Royal Society of London* 124 (1834): 247–308. doi:10.1098/rstl.1834.0017

H11 Faraday, Michael. “Experimental Researches in Electricity—Twenty-Eighth Series.” *Phil. Trans. R. Soc. Lond.* 142 (1852): 25–56. doi:10.1098/rstl.1852.0004

H12 Thomson, William. Letter to George Gabriel Stokes, 2 July 1850, postscript; Stokes, *Smith’s Prize Examination Papers*, Cambridge, 1854, Question 8.

H13 Cayley, Arthur. “A Memoir on the Theory of Matrices.” *Phil. Trans. R. Soc. Lond.* 148 (1858): 17–37. doi:10.1098/rstl.1858.0002

H14 Riemann, Bernhard. “Über die Anzahl der Primzahlen unter einer gegebenen Grösse.” *Monatsberichte der Berliner Akademie* (1859): 671–680.

H15 Maxwell, James Clerk. “A Dynamical Theory of the Electromagnetic Field.” *Phil. Trans. R. Soc. Lond.* 155 (1865): 459–512. doi:10.1098/rstl.1865.0008

H16 Riemann, Bernhard. “Über die Hypothesen, welche der Geometrie zu Grunde liegen.” *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen* 13 (1868). doi:10.3931/e-rara-49481

H17 Klein, Felix. *Vergleichende Betrachtungen über neuere geometrische Forschungen*. Erlangen: Deichert, 1872.

H18 Poincaré, Henri. "Analysis Situs." *Journal de l'École Polytechnique* 1 (1895): 1–121.

H19 Lorentz, Hendrik A. "Electromagnetic Phenomena in a System Moving with Any Velocity Smaller than that of Light." *Proc. Royal Netherlands Acad. Arts and Sciences* 6 (1904): 809–831.

H20 Minkowski, Hermann. "Raum und Zeit." Lecture, Cologne, 1908; *Jahresbericht der Deutschen Mathematiker-Vereinigung* 18 (1909): 75–88.

H21 Einstein, Albert. "Die Feldgleichungen der Gravitation." *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften* (1915): 844–847.

H22 Noether, Emmy. "Invariante Variationsprobleme." *Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Math.-Phys. Klasse* (1918): 235–257.

H23 Schrödinger, Erwin. "Quantisierung als Eigenwertproblem." *Annalen der Physik* 386 (1926): 109–139. doi:10.1002/andp.19263861802

H24 Pauli, Wolfgang. "Zur Quantenmechanik des magnetischen Elektrons." *Zeitschrift für Physik* 43 (1927): 601–623. doi:10.1007/BF01397326

H25 Dirac, P. A. M. "The Quantum Theory of the Electron." *Proc. Royal Society A* 117 (1928): 610–624. doi:10.1098/rspa.1928.0023

H26 Gödel, Kurt. "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I." *Monatshefte für Mathematik und Physik* 38 (1931): 173–198. doi:10.1007/BF01700692

H27 von Neumann, John. *Mathematische Grundlagen der Quantenmechanik*. Berlin: Julius Springer, 1932.

H28 Rosser, J. Barkley. "Extensions of Some Theorems of Gödel and Church." *Journal of Symbolic Logic* 1, no. 3 (1936): 87–91. doi:10.2307/2269028

H29 Turing, Alan M. "On Computable Numbers, with an Application to the Entscheidungsproblem." *Proc. London Mathematical Society* s2-42 (1937): 230–265. doi:10.1112/plms/s2-42.1.230

H30 Shannon, Claude E. "A Mathematical Theory of Communication." *Bell System Technical Journal* 27 (1948): 379–423; 623–656. doi:10.1002/j.1538-7305.1948.tb01338.x; doi:10.1002/j.1538-7305.1948.tb00917.x

H31 Bohm, David. "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables. I–II." *Physical Review* 85 (1952): 166–193. doi:10.1103/PhysRev.85.166; doi:10.1103/PhysRev.85.180

H32 Wittgenstein, Ludwig. *Philosophical Investigations*. Oxford: Blackwell, 1953.

H33 Aharonov, Yakir, and David Bohm. "Significance of Electromagnetic Potentials in the Quantum Theory." *Physical Review* 115 (1959): 485–491. doi:10.1103/PhysRev.115.485

H34 Mac Lane, Saunders. *Categories for the Working Mathematician*. New York: Springer, 1971.

H35 Bell, John S. "On the Einstein Podolsky Rosen Paradox." *Physics Physique Fizika* 1 (1964): 195–200. doi:10.1103/PhysicsPhysiqueFizika.1.195

H36 Clauser, John F., Michael A. Horne, Abner Shimony, and Richard A. Holt. "Proposed Experiment to Test Local Hidden-Variable Theories." *Physical Review Letters* 23 (1969): 880–884. doi:10.1103/PhysRevLett.23.880

H37 Apple Computer. Apple II brochure, 1977. Computer History Museum scan: s3data.computerhistory.org/brochures/apple.ii.1977.102637933.pdf. Wozniak, Jobs, and the Apple II: computerhistory.org/revolution/personal-computers/17/300

H38 Berry, M. V. "Quantal Phase Factors Accompanying Adiabatic Changes." *Proc. Royal Society of London A* 392 (1984): 45–57. doi:10.1098/rspa.1984.0023

B Present source coordinates

Ref.	Source record	Exact relation retained	Office in this closure	Status
P01	Physical Reality Inside Constraint	$\Pi(e_\Pi, \ell) = c = (e_\Pi, \ell, w, r)$	Contact: field carrier, light, address, receipt.	Source declaration.
P02	Motion & Difference	$\Delta_{\text{phase}} \in \text{End}(E_p)$, $\Delta_{\text{time}} \in \mathbb{R}$	Phase and time retain distinct types.	Source declaration.
P03	Memory Return & Observable Difference	base return, lifted difference, represented difference, receipt	Return admits relational difference.	Source declaration.
P04	Independent Tunability of Local Curvature and Global Holonomy in a Source-Free $SO(2)$ Gauge Field	$F^{\text{EM}} = dA$ (source-native F), holonomy, proper time	Curvature, loop phase and duration retain separate coordinates.	Source declaration.

Ref.	Source record	Exact relation retained	Office in this closure	Status
P05	Metric Mirror Symmetry: Riemann's i Machine	$Q^{\mathsf{T}}GQ = -G$, $(Q^2)^{\mathsf{T}}GQ^2 = G$, $Q^2 \neq I$	Metric root, mirror return, analytic reader source.	Exact finite / Source declaration by row.
P06	The 28 Seam Mirror Wheel	$\mathrm{Phase}_4(n+4) = \mathrm{Phase}_4(n)$, $\mathrm{Cycle}_4(n+4) = \mathrm{Cycle}_4(n) + 1$	Occurrence and history advance through phase return.	Source declaration.
P07	Horizon Silence: Viability Boundary and Zero Residual	$\Gamma_{J(p)}$, $\mathrm{Gate}_{\Gamma_{J(p)}}$, ε, θ	Typing, admission, residual, silence, survival.	Source declaration, HS.5–HS.19.
P08	Quotient	eight-address / twenty-eight-seam carrier	Packet form and quotient carrier.	Source declaration.
P09	Infinity One Zero Lift	projective mirror M_{pr}, K_{pr}-unitary chart, real adapter A_{ch}, escapement	Projective/unitary lift.	Exact finite / Source declaration by row.
P10	Mechanical Reality: Complete Transmission Edition	reader grammar and transmission record	Formal-lineage offices and reader discipline.	Source declaration.
P11	Minimum Mechanics 010a–010u	canonical, geometric, source and jurisdictional verification interfaces	Lean-compiled theorem and source verification interfaces.	Verification record; statuses in §9, §8.2 and Appendix E.
P12	Symmetric Minimum, v1.7.14 candidate; this reference edition v1.7.16	finite root, return operators, action, phase reader, residual ledger	Current coordinate register; 64-check verifier receipt; 140-check independent receipt.	Exact finite / Exact derived / Source declaration by row.
P13	Suture Field Dynamics: Regular Hamiltonian Reduction	constrained action, suture equations, regular Einstein–Maxwell reduction	Metric–flux realization.	Source declaration, SD.0–SD.2 and SD.21.
P14	Hamiltonian Suture field-dynamics continuum record	variational / Hamiltonian realization, constraint and gauge reduction, refinement and quotient, observable transport	Continuum obligations for Gravity, Light, Constraint, Horizon; separate from P13's regular reduction.	Source declaration; conditional on its stated reduction and observable conditions.
P15	The Theory of Everything: Noether–Klein Scalar Symmetry, Covariant Field Interfaces, and an Exact 2×2 Lorentzian Root, contact–record edition	$\mathcal{C}_\rho$, $B = Q^2$, reader fibres, ordered snoc receipt, N_{alg}	Contact normal form, reader resolution, historical order and quadratic-integer metric address.	Source declaration / Exact finite by displayed relation.
P16	Minimum Endurance Delta: Dual-Form Motion, Cup Symmetry, and the Positive Horizon of Return	dual-form return, Memory Return datum, non-erasing four-phase clock, finite horizon	Oriented difference, return, lift, receipt, silence and endurance.	Source declaration / Exact finite by displayed relation.
P17	The i Fold: Operator Reversal, Golden Null Transport, and Positive Return	$(\omega, J) \mapsto g_J$, golden seam, typed positive read	Positive local read and the stated arithmetic-fold interface requirement.	Exact finite / Source declaration.
P18	Linear Infinity: Return Without Erasure in a Minimum Two-Form Architecture	$p_{\mathrm{ax}} \circ L_{\mathrm{ax}} = p_{\mathrm{ax}}$, Λ-shift, MEMORY, finite 137/548 address	Quotient return, lift, compositional memory, recognition and finite address closure.	Exact derived / Source declaration by row.

Ref.	Source record	Exact relation retained	Office in this closure	Status
P19	Pianoforte i Architecture Δ	twelve-key phase, lift and history relation	Human-scale return normal form and typed operator register.	Source declaration.
P20	The Miracle of Mercy: a minimum derivation of Endurance (one page, 18 July 2026)	$U_\pi = -I$; $[-q] = [q]$; $U_\pi^{\mathsf{T}}\{I, G\}U_\pi = \{I, G\}$; $U_\pi - I = -2I \neq 0$; $I + U_\pi = 0$; $\Phi(\pi)$	Closing read of the register over Gravity, Light, Memory, Motion.	Source declaration.

C The typed coordinate register

Each row fixes the typed map, the invariant or condition, the source coordinates (H: Appendix A; P: Appendix B) and the status of one word. The one-sentence definitions are in §3.3.

#	Word	Typed map	Invariant or condition	Source coordinates	Status
1	Reader	declared carrier $V = \mathbb{R}^2$; general $\rho : X \to Y$	relation taken up at a stated resolution	H13, H14, H34, H37; P05, P15, P18	Exact finite / Source declaration.
2	Identity	retained criterion ι	what the reader carries through update as required for continuation	P03, P10	Reader source declaration.
3	Contact	$\Pi(e_\Pi, \ell) = c$; $\mathcal{C}_{\rho_{\mathrm{rec}}}$	$c = (e_\Pi, \ell, w, r)$; stated transport, reader and ordered record	H35, H36; P01, P15, P18	Source declaration; χ_Π Candidate.
4	Light	null vector k, field two-form F^{EM}, reader null lines $\mathcal{N}_G$	$g^{-1}(k, k) = 0$; $dF^{\mathrm{EM}} = 0$, $d \star_g F^{\mathrm{EM}} = j$	H11, H15, H19–H21, H37; P01, P14	Exact finite ($\mathcal{N}_G$); Interface (k through Contact).
5	Memory	$\mu_\Sigma(t)$; functor $\mathsf{MEMORY} : \mathcal{P} \to \mathcal{R}$	phase-coordinated compositional relation through which distinct occurrences remain comparable; $\mathsf{MEMORY}(\mathrm{id}_x) = \mathrm{id}_{\mathsf{MEMORY}(x)}$	H07, H18, H30, H33, H34; P03, P16, P18	Source-declared reader interface; distinct from Receipt.
6	Recognition	reader $\rho : X \to Y$, required distinction $d_{\mathrm{req}} : X \to D_{\mathrm{req}}$	$d_{\mathrm{req}} = \delta_{\mathrm{req}} \circ \rho$ for some decoder $\delta_{\mathrm{req}} : Y \to D_{\mathrm{req}}$	H18, H30, H32, H35, H36; P03	Exact derived / Source declaration.
7	Calibration	ρ_Σ with declared words, instruments, references and residuals	the read has stated comparison conditions	H04, H11–H12, H30, H32; P01, P10	Interface.
8	Metric	$g_J(u, v) = \omega(u, Jv)$; $g_G(v, w) = v^{\mathsf{T}}Gw$ on $V = \mathbb{R}^2$	$g_J(u, u) \geq 0$; $G^2 = 5I$, $G^{-1} = G/5$, $\mathrm{sig}(G) = (1, 1)$	H16, H19–H21; P05, P16, P17, P19	Exact finite.
9	Gravity	metric field $g : T\mathcal{M} \times T\mathcal{M} \to \mathbb{R}$	$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$; $G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$	H16, H19–H21; P01, P05, P14	Source declaration.
10	Phase	$U_\alpha = e^{\alpha J}$	$J = U_{\pi/2}$, $J^2 = -I$, $U_\pi = -I$, $U_{2\pi} = I$	H07, H14, H23–H25, H38; P05, P16, P17, P19	Exact finite / Exact derived.
11	Mirror	$H : V \to V$	$H^2 = I$, $HJH = J^{-1}$, $H^{\mathsf{T}}GH = G$	H04, H14, H17; P05, P16, P18, P19	Exact finite.

#	Word	Typed map	Invariant or condition	Source coordinates	Status
12	Carriage	$L_s = e^{sS}$, $S = HJ$	$S = (2Q - I)/\sqrt{5}$, $L_{\mathrm{ax}}^2 = Q^2$, $L_s^{\mathsf{T}} G L_s = G$, $H L_s H = L_{-s}$	H07, H17, H19–H20; P05, P16, P18	Exact finite / Exact derived.
13	Wheel	$F_{\mathrm{esc}} : (\mathbb{Z}/4) \times V \to (\mathbb{Z}/4) \times V$	$F_{\mathrm{esc}}^4(\varphi, z) = (\varphi, L_{\mathrm{ax}} z)$	H07, H18; P06, P16, P18	Exact finite.
14	Motion	declared state transport fl_t; finite case F_{esc}	$m\ddot{q} = -\nabla V$; $\dot{f} = \{f, H_{\mathrm{Ham}}\}$; $\dot{Q}_k = \frac{\hbar}{m_k} \mathrm{Im} \frac{\nabla_k \psi}{\psi}(Q, t)$	H06, H08, H10, H31, H37; P02, P06	Source declaration / Exact finite.
15	Scope	Σ, (ρ, i)	$\rho : X \to Y$, $\Lambda(i) = 0$, i lies on the rail	H04, H17, H32; P03, P05, P18	Exact finite / Source declaration.
16	Time	axial coordinate Λ	$\Lambda(L_t z) = \Lambda(z) + 2t$ on the rail; $T = 2 \log \Phi$	H07, H23, H25; P02, P05, P16, P18, P19	Exact derived from the declared axial coordinate and V-N4.
17	Receipt	one appended entry r_t	source, calibration, transform and residual remain available for reconstruction	H22, H30; P01, P03, P15, P16, P18, P19	Source declaration; distinct from Memory.
18	Return	base reader π_{base}	$\pi_{\mathrm{base}}(\tilde{a}\tilde{x}) = \pi_{\mathrm{base}}(\tilde{x})$	H01, H02, H07, H18, H38; P03, P16, P18	Exact derived.
19	Lift	deck action $\tilde{a}$	$\tilde{a}\tilde{x} = \gamma\tilde{x}$; $q(\xi + \varpi) = q(\xi)$ with $\xi + \varpi$ retained	H18, H38; P03, P16, P18	Exact derived.
20	Orbit	$z_n = L_{\mathrm{ax}}^n z_0$	$F_{\mathrm{esc}}^{4n}(\varphi, z_0) = (\varphi, z_n)$	H07, H18; P06, P16, P18	Exact derived.
21	Worldline	rail $\{\delta = 0\}$ with L_s	$ds_{\mathrm{ax}}^2 = -\ell_0^2 d\Lambda^2$	H20, H21; P02	Exact derived / Interface for scale.
22	Horizon	$(H_n^{\mathrm{hist}}, \mathcal{F}_n^{\mathrm{hor}}, \mathrm{Admit}_n)$	$\mathrm{Th} \nvdash \mathrm{Gd}_{\mathrm{Th}}$; $\mathrm{Admit}_n : \mathcal{F}_n^{\mathrm{hor}} \rightharpoonup \mathcal{L}_{n+1}^{\mathrm{hor}}$	H26, H28, H29; P07, P14	Source declaration.
23	Constraint	$\Gamma_{J(p)}$	$(\mathrm{Condition}, \mathrm{Constraint}, \mathrm{SourceAuthority}, \mathrm{Namespace})_{J(p)}$	H26–H29, H34; P07, P14	Source declaration.
24	Gate	$\mathrm{Gate}_{\Gamma_{J(p)}}$	$D_{\Gamma_{J(p)}} \to \{\mathrm{can}, 2\}$	H26–H29; P07	Source declaration.
25	Silence	residual $\varepsilon_{\mathrm{HS}}$	$\varepsilon_{\mathrm{HS}}(p) = 0$, $\mathrm{Silence}_{\mathrm{fit}}(p) = 1 \iff \mathrm{Complete}(p)$	H09, H12, H22; P07, P16	Source declaration.
26	Survival	$\theta_{J(p)}$	$\theta_{J(p)}(x(p)) = 1$; required invariant remains carried through admitted motion	P07, P16	Source declaration.
27	Forget	reader $\rho : X \to Y$ relative to required $d_{\mathrm{req}} : X \to D_{\mathrm{req}}$	$\exists x_1, x_2 \in X$ with $\rho(x_1) = \rho(x_2)$ and $d_{\mathrm{req}}(x_1) \neq d_{\mathrm{req}}(x_2)$; no decoder δ_{req} gives $d_{\mathrm{req}} = \delta_{\mathrm{req}} \circ \rho$	H18, H30; P03, P10	Exact derived / Source declaration.
28	Endurance	$\Phi(\pi)$	$U_\pi = -I$; $[U_\pi q] = [q]$; $U_\pi^{\mathsf{T}} G U_\pi = G$; $U_\pi - I = -2I \neq 0$; $I + U_\pi = 0$; read 0 at $[\Phi, 1]$; L_{ax} and H generate D_∞; $\binom{8}{2} = 28$	H14, H23; P05, P06, P08, P20	Exact finite / Exact derived (§11.1); closure over rows 9, 4, 5, 14: Source declaration.

D Symbol separation ledger

Symbols that share a letter keep separate offices. Each source retains its native lettering at citation.

Symbol	Office in the finite reader	Other retained office	Separation rule
G	reader metric matrix	$G_{\mu\nu}$, Einstein tensor	Tensor indices mark the field equation.
T	axial period $2\log\Phi$	$T_{\mu\nu}$, stress–energy; T_{ad}, adiabatic period; Th, a formal theory	Tensor indices mark stress–energy; subscript ad marks the adiabatic period; Th names a formal theory.
H	metric mirror	$\mathcal{H}_{\mathrm{ax}}$, $\mathcal{H}_t$, H_{Ham}, H_n^{hist}	Subscript marks the other office.
F_{esc}	four-step phase map	$F^{\mathrm{EM}} = dA$, $\mathcal{F}_n^{\mathrm{Fib}}$, $\mathcal{F}_n^{\mathrm{hor}}$	Subscript or superscript fixes the office; each source retains its native F at citation.
J	one-turn matrix	j, electromagnetic current; $J(p)$, packet jurisdiction	Bare J is the finite turn (§4); the argument form $J(p)$ fixes jurisdiction. Source-native J_{rot}, where retained, remains a source coordinate.
i_{turn}	J as the finite complex turn	i, scope point on the rail	Subscript remains only where the turn office is required beside the scope point.
φ	wheel phase address	Φ, golden ratio	Letterform and office remain distinct.
Φ_{rd}	candidate read of e-motion	Φ, golden ratio	Subscript fixes the read interface.
$\Phi(\pi)$	closing read of the register (word 28)	Φ, golden ratio; Φ_{rd}	The argument (π) always accompanies the closing read; the golden ratio carries none.
Φ^*	$1-\Phi = -\Phi^{-1}$, second root of $x^2 = x+1$	Φ^* is not an adjoint or dual	Used only in §11.1; the star marks the second root.
U_θ, U_α	turn $e^{\theta J}$, the phase family of §4; the same object under two angle letters	U_{pr}, unitary chart	An angle subscript marks a turn; "pr" marks the chart.
Ψ	golden conjugate return root	Ψ_{wr}, candidate g-memory write; $\tilde{\Psi}_{\mathrm{wr}}$, its candidate lift	Bare Ψ retains the root; subscript and tilde fix the interface offices.
Q	recurrence matrix	Q_k, Bohm configuration coordinate	Index marks the configuration coordinate.
Λ	axial coordinate	Λ_{Lor}, Lorentz transformation	Subscript marks the Lorentz map.
Λ_{cos}	cosmological constant in $\mathcal{S}_{\mathrm{SF}}$	axial coordinate Λ	Subscript marks the field-action constant.
β_t	phase angle	$\theta_{J(p)}$, survival condition	Separate Greek letters.
$\hat{C}$	internal antisymmetric form in the suture action	c, contact record	Hat marks the internal form; lower-case c marks the record.
M_{pr}	projective mirror in P09	$\mathcal{M}$, manifold	Script distinguishes the finite mirror from the manifold.

Symbol	Office in the finite reader	Other retained office	Separation rule
K_{pr}	positive form in P09	K_{tr}, trace weight in the analytic reader	Subscripts distinguish the projective chart and the analytic kernel.
A_{ch}	real chart adapter in P09	P01–P19 source labels; source-native P	The local adapter label avoids both offices.
$\varepsilon_{\mathrm{mech}}$	finite residual	$\varepsilon_{\mathrm{HS}}$, occurrence residual	Every residual remains subscripted.
ℓ	light	ℓ_0, axial metric scale	Zero marks the scale.
ρ	read $X \to Y$	ρ_t, phase reader; ρ_{hid}, hidden-variable measure	Subscript marks the phase reader and the hidden-variable measure.
x_1, x_2	witness states in X	x, y, chart coordinates or adapter scalars; $\tilde{x}$, deck point; $x(p)$, packet state	Subscripts mark witnesses; tilde marks a lifted point; an argument marks packet state.
d_{req}	required distinction $X \to D_{\mathrm{req}}$	d, exterior derivative; differentials in ds and $d\Lambda$	Subscript fixes the reader-obstruction coordinate.
D_{req}	codomain of the required distinction	$D_{\Gamma_{J(p)}}$, typed gate domain; D_4, dihedral group	Subscript fixes the codomain; the gate and the group keep their own notation.
δ_{req}	decoder $Y \to D_{\mathrm{req}}$	δ, rail coordinate	Subscript fixes the decoder; bare δ remains the rail coordinate.
L_s	signed carriage family	L_{ax}, L_{pr}, $\mathcal{L}^{\mathrm{hor}}_{n+1}$	Subscript or calligraphic form fixes the carrier.
Γ_{rot}	carriage/mirror group	$\Gamma_{J(p)}$, typing context	Subscript names the carrier.
S	axial generator HJ	S_{CHSH}, correlation sum (H36)	Subscript marks the correlation sum.
λ	analytic cell coordinate $\lambda = v + kT$	λ_{hid}, hidden variable (H35, H36)	Subscript marks the hidden variable.
γ	element of Γ_{rot}; deck element	γ_B, Berry phase (H38); γ_{ij}, spatial metric (§9)	Subscript marks the geometric phase; indices mark the spatial metric.
p	packet in $J(p)$, $\Gamma_{J(p)}$	p_{jt}, joint distribution	Subscript marks the distribution.
E	E-valued coframe fibre	E_{cor}, correlation; E_B, adiabatic energy; E^i, electric field (§9)	Subscript marks each; index marks the field.
E_e, M_g	candidate e-carriage and g-memory carriers	$\tilde{E}_e$, $\tilde{M}_g$, lifted carriers; E, coframe fibre; g, metric field carriers	Subscripts fix the candidate carriers; tildes fix their declared lifts.
A_e, B_g	candidate adapters in the read–write diagram	A_{ch}, real chart adapter; source-native B	Subscripts fix the candidate adapters.
π_e, π_g	candidate quotient maps for the lifted write	p_{ax}, axial quotient; π^{ij}, momentum (§9)	Greek π with subscript fixes the interface quotient maps; indices mark momentum.
e_1, e_2	witness elements of E_e	e_Π, embodied metric carrier; e, declared e-motion label	Subscripts mark witnesses; Π marks the field carrier.
C	$\hat{C}$, internal form; c, contact record	C_{par}, parameter loop; $C[v]$, momentum generator (§9)	Subscript marks the loop; the bracket argument marks the generator.
ω	alternating bilinear map on V	Ω, its coordinate matrix; $\Omega_{\mathcal{P}_i}$, arithmetic-interface form	Lower case names the map; the capital carries its declared matrix or carrier.
g_J	positive local read from the finite turn	G, signed reader metric; $g_{\mu\nu}$, field metric	Subscript or indices fix the finite, reader and field offices.

Symbol	Office in the finite reader	Other retained office	Separation rule
MEMORY	compositional map $\mathcal{P} \to \mathcal{R}$	μ_Σ, memory coordinate; R_{MR}, receipt	Sans serif names the map; Greek μ the coordinate; R the record.
L	ordered outcome ledger in the contact record	L_s, L_{ax}, L_{pr}, carriage; $\mathcal{L}_n^{\mathrm{Luc}}$, Lucas sequence	Sans serif marks the ledger; subscript or context fixes transport and sequence.
p_{ax}	axial quotient map	packet p, p_{lift}, joint distribution p_{jt}	Subscript fixes the quotient map.
W, W_{pno}	circle phase read; piano lifted key address	W_k, source-native key address	Subscript or source citation fixes the carrier.
Fac_G	metric factor-profile reader	F_{esc}, F^{EM}, $\mathcal{F}_n^{\mathrm{Fib}}$	The full operator name avoids the field and wheel F offices.
$\mathcal{T}_{\mathrm{arith}}$, $\mathcal{Q}_{\mathrm{arith}}$	proposed arithmetic fold map and its quadratic form	Q, recurrence matrix; $\mathcal{P}_i$, positive-read carrier	Calligraphic letters and subscripts fix the open interface.
R	$R(g)$, Ricci scalar	R_{par}, parameter point	Boldface and subscript mark the parameter point.
m	cell count; m_k, mass	m_n^{mach}, stored machine state	Superscript marks the machine state.
N, M	lapse functions (§9)	M, source-native manifold lettering in $\mathcal{S}_{\mathrm{SF}}$; M_{pr}	Context fixes the office; the manifold is $\mathcal{M}$ elsewhere.

E Lean closure: certificate and findings

This appendix carries the verification state of Minimum Mechanics 010k–010u as read for this edition. It is a record, not a proof: the proofs are the pinned Lean sources.

E.1 Provenance

Repository `Coppalini-Architecture/coppalini-architecture`, branches `codex/*`, read anonymously by `git fetch --depth 1 origin <full SHA>` (a short SHA does not resolve). Toolchain: Lean 4.33.0. For each part the following were done: the pinned source was fetched and hashed against the pin recorded in its manifest; the source was scanned for `sorry`, added axioms and `native_decide`; the exact-arithmetic script was rerun with a vacuity census; and the CI status page at the pin was read. Not done: a local Lean compile (the toolchain download is blocked in the working environment), and reading CI logs (they require sign-in). Earlier receipts are append-only.

E.2 Pins and runs

Part	Pinned commit	CI / archive	Script census
010k	`04cb6a80` (the manifest states `e35c6a22`; mismatch recorded)	not in carried record	not in carried record
010l	`f8184928`	not in carried record	not in carried record
010m	`b25f87cc`	not in carried record	not in carried record
010n	`7391bb19`	CI #10; archive #237 green	not in carried record
010o	`b63e938c…` (short form carried)	CI #8 green	261/261; conditional result

Part	Pinned commit	CI / archive	Script census
010p	`9ddd6407fa689c1c4b0a61bb353350cca9cd5598`	compiler #7, archive #263 green	643/643; pointwise
010q	`5fda559c33e9f63d12b8bb2789f88364092272f4`	CI #3 green; archive run `43880909497` unread	2602/2602
010r	`ad164c628d629cf918ba3ace959a7ad20731fc1d` (branch head; the kit gives no pin)	CI #4 green after three failed runs	2704/2704, of which 200 tautologies
010s	`372981a983c221a79784496ea4b0d92d4b57cd7f`	CI #3 green; archive run unread	—
010t	`ca6cd5bfd652316d0218fe9f47f2b26e6cd04c30`	CI #6, archive #294 green	4529/4529, of which 1800 vacuous Fourier checks
010u	`b9e939d514a63fb021592f79a316f70cbc70ed8a`	CI #5, archive #302 green; 45-module closure; 20 audits	—

Hash prefixes recorded: 010q source `3b404faf`, workflow `2dc0e6a9`; 010r Lean `4be5e512`, workflow `caa8d867`; 010t source `8a922b87`, workflow `0a6df763`; 010u source `47aeccb2`. The AxiomaticBuild README (`ecfd19cc9378a3b4`) and PDF (`9fe92b0eb7859896`) match the 010u manifest rows. Where the table says "not in carried record", the pin was verified but the further columns were not carried into the record this edition was built from.

E.3 The CI guard

The guard `scripts/verify_lean_010s.py` (010s) fails closed on a missing or empty log, on axioms outside the allowed set (`propext`, `Quot.sound`, `Classical.choice`), on a hash mismatch or duplicate, and on forbidden source tokens. Its ten tests pass in the working environment. Edge cases: a log containing a compiler `error:` or `declaration uses 'sorry'` together with an audit line passes the log check, and is caught by the compile exit code and the source scan; a `"sorry"` string literal is rejected.

E.4 Findings ledger

Classes: *drop* (a needed item absent), *add* (a claim or check with no content), *strength* (stated strength exceeds premises), *record* (a fact for the record).

#	Finding	Class	Disposition
1	010k manifest names pin `e35c6a22`; the verified pin is `04cb6a80`.	record	Pin carried here.
2	010r kit gives no pin and no receipt; verified against the branch head.	drop	Head recorded in E.2.
3	A sign slip in the 010q text.	add	Recorded.
4	010o is conditional on seven premises; 010p is pointwise only.	strength	Stated as such in §9.4.
5	CI regex arm `Lean\\.ofReduceBool` in single quotes matches a literal backslash: a dead arm in 17 workflows (010b–010q). A source sweep found no exposure.	add	Repaired in 010r and 010s.
6	Vacuous or tautological check blocks: 200 in 010r; 1800 vacuous Fourier checks and 500 structural $0 = 0$ in 010t.	add	Counted out of the evidence here.
7	010u: the history conjunct of **Admitted** is not tied to **Enough**.	drop	Open item 6 (§11).

#	Finding	Class	Disposition
8	Overview counts do not reproduce: 010q 2603 against 2602 here; 010r 2904 against 2704 here.	record	Recorded.
9	`#print axioms` lines exceed the reported matched messages: 342 against 290 (010u); 219 against 202 (010t). Consistent with axiom-free theorems printing a different message; unverified without logs.	record	Open until logs are readable.
10	010u theorems mostly unpack definitions; “universal” is definitional.	strength	Stated in §8.2.
11	`SmoothCompletion` is never instantiated; `Int` blocks a real instance.	strength	§9.5.
12	Compile evidence is the CI status page only; logs need sign-in.	record	§10.3.
13	The register ended at word 27. The half-turn read, the zero read at the fixed point and the twenty-eight reads of the source record were not carried.	drop	Closed in §11.1; open items 7 and 8 (§11).

F Build and transport

This edition is one self-contained TEX file with no external inputs.

- **Engine.** XeLaTeX. Run `xelatex symmetric_minimum_v1_7_15.tex` twice, so that the contents and cross-references settle.
- **Packages** (standard TeX Live): fontspec, amsmath, amsthm, mathtools, booktabs, longtable, array, tabularx, enumitem, xcolor, microtype, fancyhdr, tcolorbox, hyperref. Fonts: Latin Modern, bundled with TeX Live. One table cell (H01) uses a CJK font if one is installed and falls back to a romanization otherwise.
- **Independent check.** `python3 -I check_symmetric_minimum_independent.py` (needs `sympy`); it prints the per-family counts and ends with `RESULT OK`.
- **Integrity.** `SHA256SUMS` lists the checksums of the files delivered together; verify with `sha256sum -c SHA256SUMS`.
- **Companion.** The repair receipts v1.3 through v1.7.16 are in the separate Repair Ledger.