THE THEORY OF EVERYTHING

An Exact 2×2 Lorentzian Construction

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Abstract. Let

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad G = [Q, \Omega] = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}.$$

The real symmetric matrix G has Lorentzian signature. Its normalized involution $H = G/\sqrt{5}$ reverses the one-parameter group $L_s = \exp(s(2Q - I)/\sqrt{5})$:

$$L_s^{\mathsf{T}} G L_s = G, \qquad H L_s H = L_{-s}, \qquad L_{2\log\Phi} = Q^2.$$

An explicit congruence carries G to the null form $2\,du\,dv$, and $G \oplus I_2$ has signature $(3, 1)$. The construction gives a discrete variational map, a $U(1)$ phase inversion $H U_\theta H = U_\theta^{-1}$, and a finite Boltzmann entropy identity. General relativity, quantum gravity, and the Standard Model are stated through their field equations, state spaces, gauge algebra, and parameter spaces.

I. FIELD-THEORETIC REQUIREMENTS

A unified field theory contains a Lorentzian spacetime sector, a local quantum field theory of matter and gauge fields, a quantum gravitational state space, a parameter and mass spectrum, and a statistical-mechanical limit. The standard equations and conditions used in this paper are listed below.

Sector	Standard formal requirement	Mathematical result stated here
General relativity	Lorentzian metric, causal cone, curvature, and Einstein field equation	An exact signature-$(3,1)$ extension, null coordinates, observer decomposition, and pp-wave curvature formula
Quantum gravity	Physical Hilbert space, constraint quotient, continuum limit, and a propagating spin-two sector	The physical quotient and spin-two residue criterion are stated in Section VI
Standard Model	$SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry, Higgs mechanism, matter parameters, and renormalized observables	The gauge-algebra count, neutral electroweak mass matrix, tree-level mass relations, and nineteen-parameter space
Gauge phase	Connection, holonomy, and $U(1)$ representation	An exact real 2×2 representation of phase inversion
Classical mechanics	Action principle and symplectic evolution	A discrete stationary action whose update preserves the canonical symplectic form
Statistical mechanics	Microstates, macrostates, entropy, and coarse graining	The finite Boltzmann counting identity for a refinement of a macrostate

TABLE 1. Field-theoretic conditions and the equations established or specified in this paper.

II. THE TWO-DIMENSIONAL LORENTZIAN FORM

Definition II.1 (Root carrier)**.** *Let $V = \mathbb{R}^2$ and let*

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad G = Q\Omega - \Omega Q = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}. \tag{1}$$

Write

$$\Phi = \frac{1+\sqrt{5}}{2}, \qquad \Psi = \frac{1-\sqrt{5}}{2}, \qquad S = \frac{2Q-I}{\sqrt{5}}, \qquad H = \frac{G}{\sqrt{5}}, \qquad L_s = e^{sS}. \tag{2}$$

The metric read is $q_G(z) = z^{\mathsf{T}} G z$.

Theorem II.2 (Scalar-symmetry theorem)**.** *The root carrier satisfies*

$$\begin{gathered} Q^2 = Q + I, \qquad \Omega^2 = -I, \qquad G^2 = 5I, \qquad \det G = -5, \\ S^2 = I, \qquad S^{\mathsf{T}} G + GS = 0, \qquad H^2 = I, \qquad H^{\mathsf{T}} G H = G, \qquad HSH = -S, \\ Q^{\mathsf{T}} G Q = -G, \qquad (Q^2)^{\mathsf{T}} G Q^2 = G, \qquad H L_s H = L_{-s}. \end{gathered} \tag{3}$$

Moreover,

$$L_{\log \Phi} = \frac{Q + 2I}{\sqrt{5}}, \qquad L_{2\log\Phi} = Q^2. \tag{4}$$

For the four-step map

$$\mathcal{T}(p,z) = \begin{cases} (p+1,z), & p=0,1,2, \\ (0,Q^2z), & p=3, \end{cases} \qquad (p,z) \in \mathbb{Z}_4 \times V, \tag{5}$$

one has

$$\mathcal{T}^4(p,z) = (p,Q^2z), \qquad \big(p,q_G(Q^2z)\big) = \big(p,q_G(z)\big). \tag{6}$$

Proof. Direct multiplication gives the first line of (3). The identity $G^2 = 5I$ gives $\det G = -5$ and $H^2 = I$. Since

$$(2Q-I)^2 = 5I, \qquad (2Q-I)^{\mathsf{T}}G + G(2Q-I) = 0, \tag{7}$$

the second line of (3) follows. Exponentiating the infinitesimal relation in (7) gives $L_s^{\mathsf{T}} G L_s = G$. The direct identity $HSH = -S$ gives

$$HL_sH = He^{sS}H = e^{sHSH} = e^{-sS} = L_{-s}.$$

Because $S^2 = I$,

$$e^{sS} = (\cosh s)I + (\sinh s)S. \tag{8}$$

The golden identities $\Phi - \Phi^{-1} = 1$ and $\Phi + \Phi^{-1} = \sqrt{5}$ give

$$\cosh(\log\Phi) = \frac{\sqrt{5}}{2}, \qquad \sinh(\log\Phi) = \frac{1}{2}.$$

Substitution in (8) yields the first identity in (4). Squaring it and using $Q^2 = Q + I$ yields $L_{2\log\Phi} = Q^2$. Equation (6) is then immediate from the definition of $\mathcal{T}$ and $(Q^2)^{\mathsf{T}} G Q^2 = G$. □

The identities in *(3)–(4)* define the scalar-symmetry relation used in the remaining sections.

Corollary II.3 (Null rays and golden transport)**.** *The two null lines of G are spanned by*

$$v_\Phi = \begin{pmatrix} \Phi \\ 1 \end{pmatrix}, \qquad v_\Psi = \begin{pmatrix} \Psi \\ 1 \end{pmatrix}. \tag{9}$$

The matrix Q acts on them by $Qv_\Phi = \Phi v_\Phi$ and $Qv_\Psi = \Psi v_\Psi$. The even transport Q^2 preserves each null ray and the signed metric q_G.

Proof. The characteristic equation of Q is $\lambda^2 - \lambda - 1 = 0$, with roots Φ and Ψ. Direct substitution gives $v_\Phi^{\mathsf{T}} G v_\Phi = v_\Psi^{\mathsf{T}} G v_\Psi = 0$. □

III. DISCRETE VARIATIONAL MECHANICS

Let $B = Q^2$. For $z_j, \xi_j \in \mathbb{R}^2$ and an integer $N \geq 1$, consider the discrete action

$$\mathcal{A}_d[z,\xi] = \sum_{j=0}^{N-1} \xi_{j+1}^{\mathsf{T}} \big(z_{j+1} - Bz_j\big). \tag{10}$$

The variables ξ_j are covectors represented in the chosen Euclidean coordinates. With z_0 and z_N fixed, stationarity of *(10)* gives

$$z_{j+1} = Bz_j, \qquad \xi_j = B^{\mathsf{T}} \xi_{j+1}. \tag{11}$$

The full one-step map is therefore

$$\begin{pmatrix} z_{j+1} \\ \xi_{j+1} \end{pmatrix} = \mathsf{M}_{\mathrm{cot}} \begin{pmatrix} z_j \\ \xi_j \end{pmatrix}, \qquad \mathsf{M}_{\mathrm{cot}} = \begin{pmatrix} B & 0 \\ 0 & B^{-\mathsf{T}} \end{pmatrix}. \tag{12}$$

Proposition III.1 (Variational and symplectic realization)**.** *The update* (12) *is symplectic for*

$$\mathsf{J}_{\mathrm{can}} = \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}, \qquad \mathsf{M}_{\mathrm{cot}}^{\mathsf{T}} \mathsf{J}_{\mathrm{can}} \mathsf{M}_{\mathrm{cot}} = \mathsf{J}_{\mathrm{can}}, \tag{13}$$

and its configuration component preserves the Lorentzian quadratic form:

$$q_G(Bz) = q_G(z). \tag{14}$$

Proof. Equation *(11)* follows by independent variation of ξ_{j+1} and of the interior vectors z_j. Direct block multiplication proves *(13)*. Equation *(14)* is the identity $B^{\mathsf{T}} G B = G$ in *(3)*. □

IV. LORENTZIAN GEOMETRY AND CLASSICAL GRAVITY

Define

$$M = \begin{pmatrix} 1 & -\frac{1}{5} \\ -\Psi & \frac{\Phi}{5} \end{pmatrix}. \tag{15}$$

Direct multiplication gives

$$M^{\mathsf{T}} G M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \det M = \frac{1}{\sqrt{5}}. \tag{16}$$

For $z = M(u, v)^{\mathsf{T}}$, the quadratic form is consequently

$$q_G(z) = 2uv. \tag{17}$$

The two coordinate axes $u = 0$ and $v = 0$ are the null directions of this two-dimensional Lorentzian space.

The four-dimensional real form

$$\widetilde{G} = G \oplus I_2 \tag{18}$$

has signature $(3, 1)$. If $(\mathcal{M}, g)$ is a Lorentzian spacetime and $P_x \subset T_x\mathcal{M}$ is a two-dimensional Lorentzian subspace, then Sylvester's law of inertia gives an isomorphism

$$A : \mathbb{R}^2 \longrightarrow P_x, \qquad A^{\mathsf{T}} (g|_{P_x}) A = G. \tag{19}$$

In null coordinates, flat spacetime is written

$$ds^2 = 2\, du\, dv + dx^2 + dy^2. \tag{20}$$

For a future-directed unit timelike vector u_{obs} and a future-directed null vector k, define

$$g(u_{\mathrm{obs}}, u_{\mathrm{obs}}) = -1, \qquad E = -g(u_{\mathrm{obs}}, k), \qquad n = \frac{k}{E} - u_{\mathrm{obs}}. \tag{21}$$

Then

$$g(u_{\mathrm{obs}}, n) = 0, \qquad g(n, n) = 1, \qquad k = E(u_{\mathrm{obs}} + n). \tag{22}$$

Thus a null tangent vector determines an energy and a unit spatial direction relative to an observer.

The Einstein field equation, including the cosmological constant, is

$$G^{\mathrm{Ein}}_{\mu\nu} + \Lambda_{\cos} g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}, \qquad G^{\mathrm{Ein}}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{23}$$

An exact Lorentzian curvature class is provided by the Brinkmann metric

$$ds^2 = 2\,du\,dv + H_{\mathrm{pp}}(u,x,y)\,du^2 + dx^2 + dy^2, \tag{24}$$

for which, with the stated curvature convention,

$$R_{uu} = -\frac{1}{2}\left(\partial_x^2 + \partial_y^2\right) H_{\mathrm{pp}}, \qquad R = 0. \tag{25}$$

The vacuum equation is satisfied whenever H_{pp} is harmonic in (x, y). Equations *(19)–(25)* place the two-dimensional Lorentzian construction inside the standard local geometry of general relativity.

V. GAUGE PHASE AND MIRROR CONJUGATION

Let

$$\mathsf{i} = -\Omega, \qquad U_\theta = e^{\theta \mathsf{i}} = \cos\theta\, I + \sin\theta\, \mathsf{i}. \tag{26}$$

Then $\mathsf{i}^2 = -I$ and, for the complex coordinate $\chi(x,y) = x + iy$,

$$\chi U_\theta \chi^{-1}(w) = e^{i\theta} w. \tag{27}$$

The mirror equation in *(3)* gives

$$H \mathsf{i} H = -\mathsf{i}, \qquad H U_\theta H = U_{-\theta} = U_\theta^{-1}. \tag{28}$$

At $\theta = \pi$,

$$U_\pi = -I, \qquad e^{i\pi} + 1 = 0. \tag{29}$$

For a closed curve γ, electric charge q_{el}, and electromagnetic connection one-form A_{em}, the Aharonov–Bohm phase is

$$\varphi_{\mathrm{AB}} = \frac{q_{\mathrm{el}}}{\hbar} \oint_\gamma A_{\mathrm{em}}, \qquad U_\gamma = e^{-\varphi_{\mathrm{AB}} \Omega}. \tag{30}$$

Under the complex coordinate in *(27)*,

$$\chi U_\gamma \chi^{-1}(w) = e^{i\varphi_{\mathrm{AB}}} w, \qquad H U_\gamma H = U_\gamma^{-1}. \tag{31}$$

Thus the two-dimensional real representation realizes the standard $U(1)$ phase and its inverse holonomy.

VI. QUANTUM GRAVITY

For a covariant quantum-gravity model, let $\mathcal{D}_\Sigma$ be a test space and let

$$P_\Sigma : \mathcal{D}_\Sigma \longrightarrow \mathcal{D}'_\Sigma \tag{32}$$

be a rigging map. The physical Hilbert space is the completion

$$\mathcal{H}_{\Sigma}^{\mathrm{phys}} = \overline{\mathcal{D}_{\Sigma} / \ker P_{\Sigma}} \tag{33}$$

in the physical inner product. If $\mathcal{G}_{\mathrm{phys}}(k)$ is a momentum-space physical two-point function and $P_{\mathrm{TT}}(k)$ is the transverse-traceless projector, the massless spin-two residue is

$$\mathcal{W}_2[\mathcal{G}_{\mathrm{phys}}] = \lim_{k^2 \to 0} k^2 P_{\mathrm{TT}}(k) \mathcal{G}_{\mathrm{phys}}(k) P_{\mathrm{TT}}(k). \tag{34}$$

The field-theoretic condition for a propagating graviton sector is

$$\mathcal{W}_2[\mathcal{G}_{\mathrm{phys}}] \neq 0. \tag{35}$$

The physical quotient *(33)*, a continuum limit, and the nonzero residue *(35)* are the mathematical conditions that connect a covariant quantum-gravity construction to a low-energy spin-two field.

VII. STANDARD MODEL GAUGE THEORY

The gauge algebra of the Standard Model is

$$\mathfrak{g}_{\mathrm{SM}} = \mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y, \qquad \dim \mathfrak{g}_{\mathrm{SM}} = 8 + 3 + 1 = 12. \tag{36}$$

For the componentwise Lie bracket

$$\beta : \bigwedge^2 \mathfrak{g}_{\mathrm{SM}} \longrightarrow \mathfrak{g}_{\mathrm{SM}}, \tag{37}$$

the image has dimension 11: the two simple summands are generated by their brackets and $\mathfrak{u}(1)_Y$ is central. Hence

$$\dim \bigwedge^2 \mathfrak{g}_{\mathrm{SM}} = 66, \qquad \operatorname{rank} \beta = 11, \qquad \dim \ker \beta = 55. \tag{38}$$

In particular,

$$\dim \bigwedge^2 \mathfrak{su}(3) = \binom{8}{2} = 28. \tag{39}$$

For any ordered basis $(t_0, \ldots, t_7)$ of $\mathfrak{su}(3)$, the map

$$\mathfrak{a} : \mathbb{R}^{E(K_8)} \longrightarrow \bigwedge^2 \mathfrak{su}(3), \qquad \mathfrak{a}(e_{ij}) = t_i \wedge t_j \tag{40}$$

is an isomorphism, where $E(K_8)$ is the edge set of the complete graph on eight vertices. This identifies the twenty-eight edge basis vectors with the exterior-square carrier of the color algebra.

With the conventional Higgs doublet, hypercharge $Y_H = \frac{1}{2}$, and $Q_{\mathrm{em}} = T_3 + Y$, the neutral gauge-boson mass matrix is

$$\mathsf{Mass}^2_{\mathrm{neutral}} = \frac{v^2}{4} \begin{pmatrix} g_2^2 & -g_2 g_Y \\ -g_2 g_Y & g_Y^2 \end{pmatrix}. \tag{41}$$

Its eigenvalues are

$$0, \qquad \frac{v^2}{4}(g_2^2 + g_Y^2), \tag{42}$$

and the tree-level relations are

$$m_W = \frac{g_2 v}{2}, \qquad m_Z = \frac{v}{2}\sqrt{g_2^2 + g_Y^2}, \qquad m_f = \frac{y_f v}{\sqrt{2}}. \tag{43}$$

In the minimal renormalizable model with three generations, one Higgs doublet, massless neutrinos, and the strong-CP parameter retained, the continuous parameter space has nineteen coordinates:

$$\begin{aligned} &3 \text{ gauge couplings} + 2 \text{ Higgs-potential parameters} \\ &+ 9 \text{ charged-fermion masses} + 4 \text{ CKM parameters} \\ &+ 1 \text{ strong-CP parameter} = 19. \end{aligned} \tag{44}$$

An all-parameter derivation assigns every coordinate in *(44)* at a specified renormalization scale and scheme and compares the resulting observables with measurements independent of the selecting data. Neutrino masses and mixing belong to an extended parameter space.

VIII. STATISTICAL MECHANICS AND THERMODYNAMICS

Let X be a finite microstate space and let

$$\rho : X \longrightarrow \mathcal{M} \tag{45}$$

be a macrostate map. For $\mu \in \operatorname{im}\rho$, define

$$W_\rho(\mu) = |\rho^{-1}(\mu)|, \qquad S_B(\mu) = k_B \log W_\rho(\mu). \tag{46}$$

If $\eta : X \to \mathcal{R}$ is a finer macrostate map and $\rho = \lambda \circ \eta$, then every fiber $\eta^{-1}(r)$ is contained in the corresponding fiber $\rho^{-1}(\lambda(r))$. Hence

$$S_\eta(r) = S_B(\lambda(r)) - k_B \log \frac{W_\rho(\lambda(r))}{W_\eta(r)} = k_B \log W_\eta(r). \tag{47}$$

Equation *(47)* is the exact finite coarse-graining identity. It separates the entropy associated with a declared macrostate from the entropy associated with a refinement of that macrostate.

For a thermodynamic system with heat into the system and work done by the system, the energy and entropy balances are

$$\Delta U = Q_{\text{heat}} - W_{\text{out}}, \qquad dS = d_{\text{e}}S + d_{\text{i}}S, \qquad d_{\text{i}}S \geq 0. \tag{48}$$

The balances in *(48)* specify the physical boundary and process to which an entropy calculation applies.

IX. CONCLUSION

The scalar-symmetry theorem gives a complete finite result:

$$G = [Q, \Omega], \qquad G^2 = 5I, \qquad L_s^{\mathsf{T}} G L_s = G, \qquad H L_s H = L_{-s}, \qquad L_{2\log\Phi} = Q^2. \tag{49}$$

The same real 2×2 carrier has a discrete variational realization, null coordinates, a signature-$(3,1)$ extension, and a faithful representation of $U(1)$ phase inversion. The Standard Model, quantum gravity, and thermodynamic sectors enter through their native gauge, state-space, field-equation, and entropy structures. Equations *(23)*, *(33)–(35)*, *(36)–(44)*, and *(46)–(48)* state the corresponding field-theoretic requirements in the same record as the proved scalar relation.

A. THE QUADRATIC INTEGER FORM

For $x = a + b\Phi \in \mathbb{Z}[\Phi]$, define

$$N_{\text{alg}}(x) = a^2 + ab - b^2, \qquad z = \begin{pmatrix} b \\ a \end{pmatrix}. \tag{A1}$$

Then

$$z^{\mathsf{T}} G z = 2N_{\text{alg}}(x). \tag{A2}$$

Multiplication by Φ sends (a, b) to $(b, a + b)$, and consequently

$$N_{\text{alg}}(\Phi x) = -N_{\text{alg}}(x), \qquad N_{\text{alg}}(\Phi^2 x) = N_{\text{alg}}(x). \tag{A3}$$

Equations *(A2)–(A3)* are the integer form of the sign reversal under Q and the isometry under Q^2.

B. TWENTY-EIGHT EDGES

Write the vertex set of K_8 as $\{\infty\} \cup \mathbb{Z}_7$. For $r \in \mathbb{Z}_7$, let

$$M_r = \{\{\infty, r\}\} \cup \big\{\{r + j, r - j\} : j = 1, 2, 3\big\}. \tag{A4}$$

Each M_r is a perfect matching with four edges. The seven matchings partition the twenty-eight edges of K_8. Indeed, an edge incident to ∞ determines r, and an edge $\{a, b\} \subset \mathbb{Z}_7$ occurs in the matching with

$$r = 4(a + b) \pmod 7. \tag{A5}$$

Thus every edge occurs exactly once.

C. FINITE ARITHMETIC

The Fibonacci and Lucas sequences give

$$L_5^2 + 4^2 = 11^2 + 4^2 = 137 = 1 + 4F_9. \tag{A6}$$

In the finite field $\mathbb{F}_{137}$,

$$4^{17} \equiv -1 \pmod{137}, \qquad \operatorname{ord}_{137}(4) = 34, \qquad \mathbb{F}_{137}^{\times} / \langle 4 \rangle \cong C_4. \tag{A7}$$

These are exact finite-field identities.

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