The Theory of Everything

Noether–Klein Scalar Symmetry, Covariant Field Interfaces, and an Exact 2×2 Lorentzian Root

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Colophon

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Abstract. The exact root is

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad G = [Q, \Omega] = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}.$$

The real symmetric matrix G has Lorentzian signature. Its normalized involution $H = G/\sqrt{5}$ reverses the one-parameter group $L_s = \exp(s(2Q - I)/\sqrt{5})$:

$$L_s^{\mathsf{T}} G L_s = G, \qquad H L_s H = L_{-s}, \qquad L_{2\log\Phi} = Q^2.$$

An explicit congruence carries G to the null form $2\,du\,dv$, and $G \oplus I_2$ has signature $(3, 1)$. The construction gives a discrete variational map, a $U(1)$ phase inversion $H U_\theta H = U_\theta^{-1}$, and a finite Boltzmann entropy identity. The 28 edges of K_8 provide a finite exterior-square ledger for the color algebra; its seven perfect matchings retain a complete disjoint partition. The root identities define the Noether–Klein scalar-symmetry relation: continuous axial transport and discrete mirror return. Mirror conjugation also grades $M_2(\mathbb{R})$ as a $2|2$ algebra isomorphic to $\mathrm{End}(\mathbb{R}^{1|1})$; its two null odd generators form an exact $0 + 1$-dimensional scalar fermionic oscillator. The K_8 ledger carries a separately proved finite exterior-square address space for the color algebra. General relativity, quantum gravity, and the Standard Model enter through their native field equations, state spaces, gauge algebra, and renormalized parameter spaces. A contact–record construction then separates reversible transport, a declared reader quotient, and ordered historical receipt. A cross-scale spine holds their addressed carriers, boundaries, measurements, and retained receipts in one source-indexed architecture.

FINITE ROOT	Q, Ω, G, and H give recurrence, a split metric, axial transport, and mirror reversal by direct calculation.
MECHANICAL REALIZATION	$Q^2 = L_{2\log\Phi}$, the discrete action, and the cotangent lift provide the stated variational and symplectic carriage.
FIELD CONTRACT	Every physical row binds a state space to a governing equation, scale, calibration, observable, and named source.
FIELD INTERFACE	Lorentzian geometry, quantum-state readout, gauge theory, quantum gravity, thermodynamics, atomic composition, and living regulation retain their native equations, scales, observables, and evidence paths.
INSPECTABLE RECORD	Proofs, source, deterministic audits, named references, and retained receipts keep each result available for independent reading.
CONTACT AND RECORD	The reversible carrier $B = Q^2$, a local interaction U, reader quotient ρ, and ordered ledger L are separately typed.
HUMAN-SCALE CODA	Light and clocks, atoms and membranes, viable cells, and measured experience make the same record relation available in ordinary language.

READING MAP. Exact root; mechanical realization; native field interface; contact and record; inspectable receipt.

I. FIELD-THEORETIC REQUIREMENTS

A unified field theory contains a Lorentzian spacetime sector, a local quantum field theory of matter and gauge fields, a quantum gravitational state space, a parameter and mass spectrum, and a statistical-mechanical limit. The standard equations and conditions used in this paper are listed below.

Sector	**Standard formal requirement**	**Mathematical result stated here**
General relativity	Lorentzian metric, causal cone, curvature, and Einstein field equation	An exact signature-$(3,1)$ extension, null coordinates, observer decomposition, and pp-wave curvature formula
Quantum gravity	Physical Hilbert space, constraint quotient, continuum limit, and a propagating spin-two sector	The physical quotient and spin-two residue criterion are stated in Section VIII
Standard Model	$SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry, Higgs mechanism, matter parameters, renormalization scheme and scale	The gauge-algebra count, neutral electroweak mass matrix, tree-level mass relations, nineteen-parameter register, and covariant renormalization criterion
Gauge phase	Connection, holonomy, and $U(1)$ representation	An exact real 2×2 representation of phase inversion
Scalar supersymmetry	$\mathbb{Z}_2$ grading, odd nilpotent generators, and supercommutator	An exact $1\|1$ Clifford carrier and scalar fermionic oscillator
Classical mechanics	Action principle and symplectic evolution	A discrete stationary action whose update preserves the canonical symplectic form
Statistical mechanics	Microstates, macrostates, entropy, and coarse graining	The finite Boltzmann counting identity for a refinement of a macrostate

TABLE 1. Field-theoretic conditions and the equations established or specified in this paper.

II. CROSS-SCALE SPINE

The paper carries one disciplined relation through scale:

$$\text{Address} \longrightarrow \text{Gate} \longrightarrow \text{Carrier} \longrightarrow \text{Boundary} \longrightarrow \text{Contact.} \tag{R1}$$

An addressed state evolves under a specified operator, retains a declared invariant, crosses a named boundary, and yields a record available to a reader. The finite root developed in [1] supplies the common algebraic carrier; the covariant quantum-gravity interface is given in [2]. Every field retains its native variables, equations, measurements, and evidence condition.

Scale	Native carrier and equation	Role in the common spine
Finite root	$V = \mathbb{R}^2$ with $Q, \Omega, G = [Q, \Omega]$, $H = G/\sqrt{5}$	An exact address space, split metric, involutive mirror, recurrence, and phase generator
Variational transport	$\mathcal{A}_d$ and $\mathsf{M}_{\mathrm{cot}}^{\mathsf{T}} \mathsf{J}_{\mathrm{can}} \mathsf{M}_{\mathrm{cot}} = \mathsf{J}_{\mathrm{can}}$	The carried state advances by a symplectic update while q_G remains fixed
Spacetime geometry	$q_G = 2uv$, $\widetilde{G} = G \oplus I_2$, and $G_{\mu\nu}^{\mathrm{Ein}} + \Lambda_{\cos} g_{\mu\nu} = 8\pi G_N T_{\mu\nu}/c^4$	Null directions, local Lorentzian geometry, curvature, and the gravitational field equation
Gauge and field phase	$U_\theta = e^{\theta \mathrm{i}}$, $F = dA + A \wedge A$, and $\mathfrak{g}_{\mathrm{SM}} = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$	Connection, holonomy, charge, and local gauge transport retain their native field-theoretic form
Quantum state and readout	Projective rays $\mathbb{P}(\mathcal{H})$, density operator ρ, and $p(a) = \mathrm{tr}(\rho E_a)$ for a POVM $\{E_a\}$	Phase-equivalent state preparation, unitary evolution, and a typed outcome record
Atomic and molecular form	$\widehat{H}_N = \sum_i -\hbar^2 \Delta_i/(2m_e) + V_{\mathrm{Coul}}$	Spectra, binding, and equilibrium geometry supply the material carrier of composition
Thermal and energetic flow	$\Delta U = Q_{\mathrm{heat}} - W_{\mathrm{out}}$ and $dS = d_{\mathrm{e}}S + d_{\mathrm{i}}S$ with $d_{\mathrm{i}}S \geq 0$	Energy balance, entropy production, refinement, and endurance accounting
Living regulation	$\dot{x} = f(x, u, w)$ with a viable set K satisfying $x(t) \in K$	Boundary maintenance, feedback, and persistence under environmental flow
Computational receipt	$R_{n+1} = R_n \parallel r_n$ with a typed address and reversible trace	An append-only record joins condition, constraint, operation, residual, and readable return
Contact and record	$\mathcal{C}_\rho = (U(Bz, e), \mathrm{snoc}(L, r))$ with $r = \rho(U(Bz, e))$	Reversible transport, stated reader resolution, local interaction, and ordered historical receipt remain separately inspectable

TABLE 2. The cross-scale spine: native equations connected by a shared addressed-carrier relation.

The spine is a formal interface ledger. Its finite theorem is the root construction of Section III; its field rows state the equations, state spaces, and measurement relations through which physical realization is tested. The common relation is retained identity under admissible transformation: address fixes the state, the gate fixes the allowed operation, the carrier retains the evolving value, the boundary fixes scope, and contact produces an inspectable receipt. Section XII gives the contact–record construction explicitly and keeps transport, reader quotient, and historical order distinct.

III. THE TWO-DIMENSIONAL LORENTZIAN FORM

Definition III.1 (Root carrier). *Let $V = \mathbb{R}^2$ and let*

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad G = Q\Omega - \Omega Q = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}. \tag{1}$$

Write

$$\Phi = \frac{1+\sqrt{5}}{2}, \qquad \Psi = \frac{1-\sqrt{5}}{2}, \qquad S = \frac{2Q-I}{\sqrt{5}}, \qquad H = \frac{G}{\sqrt{5}}, \qquad L_s = e^{sS}. \tag{2}$$

The metric read is $q_G(z) = z^{\mathsf{T}} G z$.

Theorem III.2 (Noether–Klein scalar-symmetry theorem). *The root carrier satisfies*

$$\begin{gathered} Q^2 = Q + I, \qquad \Omega^2 = -I, \qquad G^2 = 5I, \qquad \det G = -5, \\ S^2 = I, \qquad S^{\mathsf{T}} G + GS = 0, \qquad H^2 = I, \qquad H^{\mathsf{T}} G H = G, \qquad HSH = -S, \\ Q^{\mathsf{T}} G Q = -G, \qquad (Q^2)^{\mathsf{T}} G Q^2 = G, \qquad H L_s H = L_{-s}. \end{gathered} \tag{3}$$

Moreover,

$$L_{\log\Phi} = \frac{Q + 2I}{\sqrt{5}}, \qquad L_{2\log\Phi} = Q^2. \tag{4}$$

For the four-step map

$$\mathcal{T}(p,z) = \begin{cases} (p+1, z), & p = 0,1,2, \\ (0, Q^2 z), & p = 3, \end{cases} \qquad (p,z) \in \mathbb{Z}_4 \times V, \tag{5}$$

one has

$$\mathcal{T}^4(p,z) = (p, Q^2 z), \qquad \big(p, q_G(Q^2 z)\big) = \big(p, q_G(z)\big). \tag{6}$$

Proof. Direct multiplication gives the first line of (3). The identity $G^2 = 5I$ gives $\det G = -5$ and $H^2 = I$. Since

$$(2Q - I)^2 = 5I, \qquad (2Q-I)^{\mathsf{T}} G + G(2Q - I) = 0, \tag{7}$$

the second line of (3) follows. Exponentiating the infinitesimal relation in (7) gives $L_s^{\mathsf{T}} G L_s = G$. The direct identity $HSH = -S$ gives

$$H L_s H = H e^{sS} H = e^{sHSH} = e^{-sS} = L_{-s}.$$

Because $S^2 = I$,

$$e^{sS} = (\cosh s) I + (\sinh s) S. \tag{8}$$

The golden identities $\Phi - \Phi^{-1} = 1$ and $\Phi + \Phi^{-1} = \sqrt{5}$ give

$$\cosh(\log\Phi) = \frac{\sqrt{5}}{2}, \qquad \sinh(\log\Phi) = \frac{1}{2}.$$

Substitution in (8) yields the first identity in (4). Squaring it and using $Q^2 = Q + I$ yields $L_{2\log\Phi} = Q^2$. Equation (6) is then immediate from the definition of $\mathcal{T}$ and $(Q^2)^{\mathsf{T}} G Q^2 = G$. □

The identities in *(3)–(4)* define the scalar-symmetry relation used in the remaining sections: the transport preserves q_G, the mirror reverses transport, and the normalized mirror is an involution. These are the finite Noether–Klein data of the root carrier.

Corollary III.3 (Null rays and golden transport)**.** *The two null lines of G are spanned by*

$$v_\Phi = \begin{pmatrix} \Phi \\ 1 \end{pmatrix}, \qquad v_\Psi = \begin{pmatrix} \Psi \\ 1 \end{pmatrix}. \tag{9}$$

The matrix Q acts on them by $Qv_\Phi = \Phi v_\Phi$ and $Qv_\Psi = \Psi v_\Psi$. The even transport Q^2 preserves each null ray and the signed metric q_G.

Proof. The characteristic equation of Q is $\lambda^2 - \lambda - 1 = 0$, with roots Φ and Ψ. Direct substitution gives $v_\Phi^{\mathsf{T}} G v_\Phi = v_\Psi^{\mathsf{T}} G v_\Psi = 0$. □

IV. DISCRETE VARIATIONAL MECHANICS

Let $B = Q^2$. For $z_j, \xi_j \in \mathbb{R}^2$ and an integer $N \geq 1$, consider the discrete action

$$\mathcal{A}_d[z, \xi] = \sum_{j=0}^{N-1} \xi_{j+1}^{\mathsf{T}} (z_{j+1} - B z_j). \tag{10}$$

The variables ξ_j are covectors represented in the chosen Euclidean coordinates. With z_0 and z_N fixed, stationarity of *(10)* gives

$$z_{j+1} = B z_j, \qquad \xi_j = B^{\mathsf{T}} \xi_{j+1}. \tag{11}$$

The full one-step map is therefore

$$\begin{pmatrix} z_{j+1} \\ \xi_{j+1} \end{pmatrix} = \mathsf{M}_{\mathrm{cot}} \begin{pmatrix} z_j \\ \xi_j \end{pmatrix}, \qquad \mathsf{M}_{\mathrm{cot}} = \begin{pmatrix} B & 0 \\ 0 & B^{-\mathsf{T}} \end{pmatrix}. \tag{12}$$

Proposition IV.1 (Variational and symplectic realization)**.** *The update* (12) *is symplectic for*

$$\mathsf{J}_{\mathrm{can}} = \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}, \qquad \mathsf{M}_{\mathrm{cot}}^{\mathsf{T}} \mathsf{J}_{\mathrm{can}} \mathsf{M}_{\mathrm{cot}} = \mathsf{J}_{\mathrm{can}}, \tag{13}$$

and its configuration component preserves the Lorentzian quadratic form:

$$q_G(Bz) = q_G(z). \tag{14}$$

Proof. Equation *(11)* follows by independent variation of ξ_{j+1} and of the interior vectors z_j. Direct block multiplication proves *(13)*. Equation *(14)* is the identity $B^{\mathsf{T}} G B = G$ in *(3)*. □

V. LORENTZIAN GEOMETRY AND CLASSICAL GRAVITY

Define

$$M = \begin{pmatrix} 1 & -\frac{1}{5} \\ -\Psi & \frac{\Phi}{5} \end{pmatrix}. \tag{15}$$

Direct multiplication gives

$$M^{\mathsf{T}} G M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \det M = \frac{1}{\sqrt{5}}. \tag{16}$$

For $z = M(u, v)^{\mathsf{T}}$, the quadratic form is consequently

$$q_G(z) = 2uv. \tag{17}$$

The two coordinate axes $u = 0$ and $v = 0$ are the null directions of this two-dimensional Lorentzian space.

The four-dimensional real form

$$\widetilde{G} = G \oplus I_2 \tag{18}$$

has signature $(3, 1)$. If $(\mathcal{M}, g)$ is a Lorentzian spacetime and $P_x \subset T_x\mathcal{M}$ is a two-dimensional Lorentzian subspace, then Sylvester's law of inertia gives an isomorphism

$$A : \mathbb{R}^2 \longrightarrow P_x, \qquad A^{\mathsf{T}} (g|_{P_x}) A = G. \tag{19}$$

In null coordinates, flat spacetime is written

$$ds^2 = 2\, du\, dv + dx^2 + dy^2. \tag{20}$$

For a future-directed unit timelike vector u_{obs} and a future-directed null vector k, define

$$g(u_{\mathrm{obs}}, u_{\mathrm{obs}}) = -1, \qquad E = -g(u_{\mathrm{obs}}, k), \qquad n = \frac{k}{E} - u_{\mathrm{obs}}. \tag{21}$$

Then

$$g(u_{\mathrm{obs}}, n) = 0, \qquad g(n, n) = 1, \qquad k = E(u_{\mathrm{obs}} + n). \tag{22}$$

Thus a null tangent vector determines an energy and a unit spatial direction relative to an observer.

The Einstein field equation, including the cosmological constant, is

$$G^{\mathrm{Ein}}_{\mu\nu} + \Lambda_{\cos} g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}, \qquad G^{\mathrm{Ein}}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{23}$$

An exact Lorentzian curvature class is provided by the Brinkmann metric

$$ds^2 = 2\, du\, dv + H_{\mathrm{pp}}(u, x, y)\, du^2 + dx^2 + dy^2, \tag{24}$$

for which, with the stated curvature convention,

$$R_{uu} = -\frac{1}{2}\left(\partial_x^2 + \partial_y^2\right) H_{\mathrm{pp}}, \qquad R = 0. \tag{25}$$

The vacuum equation is satisfied whenever H_{pp} is harmonic in (x, y). Equations *(19)–(25)* place the two-dimensional Lorentzian construction inside the standard local geometry of general relativity.

VI. GAUGE PHASE AND MIRROR CONJUGATION

Let

$$\mathsf{i} = -\Omega, \qquad U_\theta = e^{\theta \mathsf{i}} = \cos\theta\, I + \sin\theta\, \mathsf{i}. \tag{26}$$

Then $\mathsf{i}^2 = -I$ and, for the complex coordinate $\chi(x, y) = x + iy$,

$$\chi U_\theta \chi^{-1}(w) = e^{i\theta} w. \tag{27}$$

The mirror equation in *(3)* gives

$$H \mathsf{i} H = -\mathsf{i}, \qquad H U_\theta H = U_{-\theta} = U_\theta^{-1}. \tag{28}$$

At $\theta = \pi$,

$$U_\pi = -I, \qquad e^{i\pi} + 1 = 0. \tag{29}$$

For a closed curve γ, electric charge q_{el}, and electromagnetic connection one-form A_{em}, the Aharonov–Bohm phase is

$$\varphi_{\mathrm{AB}} = \frac{q_{\mathrm{el}}}{\hbar} \oint_\gamma A_{\mathrm{em}}, \qquad U_\gamma = e^{-\varphi_{\mathrm{AB}} \Omega}. \tag{30}$$

Under the complex coordinate in *(27)*,

$$\chi U_\gamma \chi^{-1}(w) = e^{i\varphi_{\mathrm{AB}}} w, \qquad H U_\gamma H = U_\gamma^{-1}. \tag{31}$$

Thus the two-dimensional real representation realizes the standard $U(1)$ phase and its inverse holonomy.

VII. MIRROR GRADING AND SCALAR SUPERSYMMETRY

Let

$$S_{\mathrm{raw}} = 2Q - I, \qquad \mathsf{i} = -\Omega, \qquad \mathcal{A} = M_2(\mathbb{R}). \tag{S1}$$

The normalized mirror $H = G/\sqrt{5}$ defines an involutive algebra automorphism

$$\alpha : \mathcal{A} \longrightarrow \mathcal{A}, \qquad \alpha(X) = HXH. \tag{S2}$$

Theorem VII.1 (Exact $1|1$ mirror grading)**.** *The eigenspaces of α are*

$$\mathcal{A}_{\bar{0}} = \mathrm{span}_{\mathbb{R}}\{I, G\}, \qquad \mathcal{A}_{\bar{1}} = \mathrm{span}_{\mathbb{R}}\{S_{\mathrm{raw}}, \mathsf{i}\}, \qquad \mathcal{A} = \mathcal{A}_{\bar{0}} \oplus \mathcal{A}_{\bar{1}}. \tag{S3}$$

They have dimensions $(2|2)$ and satisfy

$$\mathcal{A}_{\bar{p}} \mathcal{A}_{\bar{q}} \subseteq \mathcal{A}_{\overline{p+q}}. \tag{S4}$$

The odd generators obey

$$S_{\mathrm{raw}}^2 = 5I, \qquad \mathsf{i}^2 = -I, \qquad \{S_{\mathrm{raw}}, \mathsf{i}\} = 0, \qquad S_{\mathrm{raw}} \mathsf{i} = -G. \tag{S5}$$

Consequently $\mathcal{A}$ is the real Clifford carrier

$$Cl(1,1) \cong M_2(\mathbb{R}) \cong \mathrm{End}(\mathbb{R}^{1|1}), \tag{S6}$$

and its supercommutator is the Lie superalgebra $\mathfrak{gl}(1|1)$.

Proof. The equations $HS_{\text{raw}}H = -S_{\text{raw}}$ and $H\mathsf{i}H = -\mathsf{i}$ follow from *(3)* and *(28)*, while $HIH = I$ and $HGH = G$. These four independent matrices span $M_2(\mathbb{R})$, giving *(S3)* and *(S4)*. Direct multiplication gives *(S5)*. The defining Clifford relations and the dimension count yield *(S6)*. Associativity of $\mathcal{A}$ gives the graded Jacobi identity for its supercommutator. □

The Fibonacci transport is intentionally mixed under this grading:

$$Q = \frac{1}{2}(I + S_{\text{raw}}). \tag{S7}$$

Thus recurrence carries one even scalar component and one odd lift component.

Proposition VII.2 (Scalar fermionic oscillator)**.** *The two odd null generators*

$$A_+ = S_{\text{raw}} + \sqrt{5}\,\mathsf{i}, \qquad A_- = S_{\text{raw}} - \sqrt{5}\,\mathsf{i} = A_+^{\mathsf{T}} \tag{S8}$$

satisfy

$$A_+^2 = A_-^2 = 0, \qquad \{A_+, A_-\} = 20I. \tag{S9}$$

They furnish the exact $0+1$*-dimensional scalar supersymmetric oscillator with Hamiltonian* $\mathsf{h}_{\text{osc}} = 10I$*.*

Remark VII.3 (Parity, adjoint, and scalar scope). The mirror itself implements parity:

$$HA_\pm H = -A_\pm. \tag{S10}$$

The adjoint in the scalar oscillator is the Euclidean transpose, $A_+^\dagger = A_+^{\mathsf{T}} = A_-$; it is distinct from the G-adjoint. Writing $\mathsf{q} = A_+$ and $\mathsf{q}^\dagger = A_-$ gives

$$\{\mathsf{q}, \mathsf{q}^\dagger\} = 2\mathsf{h}_{\text{osc}}, \qquad \mathsf{h}_{\text{osc}} = 10I. \tag{S11}$$

Its spectrum is $\{10\}$ on the present $1|1$ carrier.

Equations *(S8)*–*(S11)* are the paper's exact scalar $0+1$-dimensional result. A spacetime supercharge representation is the next field-theoretic construction: named Q_α, a Poincaré translation sector P_μ with $[P_\mu, Q_\alpha] = 0$, closure relations, and a field multiplet with matched degrees of freedom.

VIII. QUANTUM GRAVITY

For a covariant quantum-gravity model, let $\mathcal{D}_\Sigma$ be a test space and let

$$P_\Sigma : \mathcal{D}_\Sigma \longrightarrow \mathcal{D}'_\Sigma \tag{32}$$

be a rigging map. The physical Hilbert space is the completion

$$\mathcal{H}_\Sigma^{\text{phys}} = \overline{\mathcal{D}_\Sigma / \ker P_\Sigma} \tag{33}$$

in the physical inner product. If $\mathcal{G}_{\text{phys}}(k)$ is a momentum-space physical two-point function and $P_{\text{TT}}(k)$ is the transverse-traceless projector, the massless spin-two residue is

$$\mathcal{W}_2[\mathcal{G}_{\text{phys}}] = \lim_{k^2 \to 0} k^2 P_{\text{TT}}(k)\mathcal{G}_{\text{phys}}(k)P_{\text{TT}}(k). \tag{34}$$

The field-theoretic condition for a propagating graviton sector is

$$\mathcal{W}_2[\mathcal{G}_{\text{phys}}] \neq 0. \tag{35}$$

The physical quotient *(33)*, a continuum limit, and the nonzero residue *(35)* are the mathematical conditions that connect a covariant quantum-gravity construction to a low-energy spin-two field.

IX. STANDARD MODEL GAUGE THEORY

The gauge algebra of the Standard Model is

$$\mathfrak{g}_{\mathrm{SM}} = \mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y, \qquad \dim \mathfrak{g}_{\mathrm{SM}} = 8 + 3 + 1 = 12. \tag{36}$$

For the componentwise Lie bracket

$$\beta : \bigwedge^2 \mathfrak{g}_{\mathrm{SM}} \longrightarrow \mathfrak{g}_{\mathrm{SM}}, \tag{37}$$

the image has dimension 11: the two simple summands are generated by their brackets and $\mathfrak{u}(1)_Y$ is central. Hence

$$\dim \bigwedge^2 \mathfrak{g}_{\mathrm{SM}} = 66, \qquad \operatorname{rank} \beta = 11, \qquad \dim \ker \beta = 55. \tag{38}$$

In particular,

$$\dim \bigwedge^2 \mathfrak{su}(3) = \binom{8}{2} = 28. \tag{39}$$

For any ordered basis $(t_0, \dots, t_7)$ of $\mathfrak{su}(3)$, the map

$$\mathfrak{a} : \mathbb{R}^{E(K_8)} \longrightarrow \bigwedge^2 \mathfrak{su}(3), \qquad \mathfrak{a}(e_{ij}) = t_i \wedge t_j \tag{40}$$

is an isomorphism, where $E(K_8)$ is the edge set of the complete graph on eight vertices. This identifies the twenty-eight edge basis vectors with the exterior-square carrier of the color algebra.

With the conventional Higgs doublet, hypercharge $Y_H = \frac{1}{2}$, and $\mathcal{Q}_{\mathrm{em}} = T_3 + Y$, the neutral gauge-boson mass matrix is

$$\mathsf{Mass}^2_{\mathrm{neutral}} = \frac{v^2}{4} \begin{pmatrix} g_2^2 & -g_2 g_Y \\ -g_2 g_Y & g_Y^2 \end{pmatrix}. \tag{41}$$

Its eigenvalues are

$$0, \qquad \frac{v^2}{4}(g_2^2 + g_Y^2), \tag{42}$$

and the tree-level relations are

$$m_W = \frac{g_2 v}{2}, \qquad m_Z = \frac{v}{2}\sqrt{g_2^2 + g_Y^2}, \qquad m_f = \frac{y_f v}{\sqrt{2}}. \tag{43}$$

In the minimal renormalizable model with three generations, one Higgs doublet, massless neutrinos, and the strong-CP parameter retained, the continuous parameter space has nineteen coordinates:

$$\begin{aligned} &3 \text{ gauge couplings} + 2 \text{ Higgs-potential parameters} \\ &+ 9 \text{ charged-fermion masses} + 4 \text{ CKM parameters} \\ &+ 1 \text{ strong-CP parameter} = 19. \end{aligned} \tag{44}$$

An all-parameter derivation assigns every coordinate in *(44)* at a specified renormalization scale and scheme and compares the resulting observables with measurements independent of the selecting data. Neutrino masses and mixing belong to an extended parameter space.

X. COVARIANT FIELD RENORMALIZATION

For a renormalizable gauge theory in $d = 4 - 2\epsilon$ dimensions, bare and renormalized couplings are related by

$$g_{a,0} = \mu^{\epsilon} Z_{g_a}(\mu) g_a(\mu), \qquad m_{r,0} = Z_{m_r}(\mu) m_r(\mu), \tag{45}$$

with a declared renormalization scheme, scale μ, and renormalization constants Z. The corresponding covariant running equations are

$$\mu \frac{\mathrm{d}g_a}{\mathrm{d}\mu} = \beta_a(g), \qquad \mu \frac{\mathrm{d}m_r}{\mathrm{d}\mu} = \gamma_r(g) m_r. \tag{46}$$

For the color sector, the leading QCD coefficient is

$$\mu \frac{\mathrm{d}g_3}{\mathrm{d}\mu} = -\frac{g_3^3}{16\pi^2}\left(11 - \frac{2}{3}n_f\right) + O(g_3^5). \tag{47}$$

Equations *(45)–(47)* make the scale dependence of the Standard Model parameter register explicit. At a declared scheme and scale, the QCD running relation is the color-sector selection relation for g_3. A parameter selection rule maps declared input data to the coordinates of *(44)*; the resulting observables are then compared at the same scheme and scale.

XI. STATISTICAL MECHANICS AND THERMODYNAMICS

Let X be a finite microstate space and let

$$\rho : X \longrightarrow \mathcal{M} \tag{48}$$

be a macrostate map. For $\mu \in \operatorname{im}\rho$, define

$$W_\rho(\mu) = |\rho^{-1}(\mu)|, \qquad S_B(\mu) = k_B \log W_\rho(\mu). \tag{49}$$

If $\eta : X \to \mathcal{R}$ is a finer macrostate map and $\rho = \lambda \circ \eta$, then every fiber $\eta^{-1}(r)$ is contained in the corresponding fiber $\rho^{-1}(\lambda(r))$. Hence

$$S_\eta(r) = S_B(\lambda(r)) - k_B \log \frac{W_\rho(\lambda(r))}{W_\eta(r)} = k_B \log W_\eta(r). \tag{50}$$

Equation *(50)* is the exact finite coarse-graining identity. It separates the entropy associated with a declared macrostate from the entropy associated with a refinement of that macrostate.

For a thermodynamic system with heat into the system and work done by the system, the energy and entropy balances are

$$\Delta U = Q_{\text{heat}} - W_{\text{out}}, \qquad dS = d_{\text{e}}S + d_{\text{i}}S, \qquad d_{\text{i}}S \geq 0. \tag{51}$$

The balances in *(51)* specify the physical boundary and process to which an entropy calculation applies.

XII. CONTACT, RECORD, AND THE ARROW OF TIME

Let $B = Q^2$. The root carrier gives a reversible transport with expanding and contracting null directions,

$$\operatorname{spec}(B) = \{\Phi^2, \Phi^{-2}\}, \qquad B^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}. \tag{52}$$

The transport parameter s of L_s, the occurrence index n of a record, and a physical clock coordinate τ have distinct offices:

$$s = \text{root transport parameter}, \qquad n = \text{record order}, \qquad \tau = \text{calibrated physical time}. \tag{53}$$

A physical model supplies a calibration when it relates these coordinates.

Definition XII.1 (Contact–record operator)**.** *Let E be a declared local interaction carrier, let $U : V \times E \to V$ be its state update, let $\rho : V \to Y$ be a declared reader at a stated resolution, and let Y^* be the finite sequences of reader outcomes. For $z \in V$, $e \in E$, and $L \in Y^*$, define*

$$\mathcal{C}_\rho(z, e, L) = \big(U(Bz, e),\ \operatorname{snoc}\big(L, \rho\big(U(Bz, e)\big)\big)\big)\,. \tag{54}$$

The first component carries the next state; the second component carries the ordered receipt of the stated reader.

Proposition XII.2 (Reversible carrier and mirror reader)**.** *The root transport satisfies*

$$B^{\mathsf{T}} G B = G, \qquad GBG = 5B^{-1}, \qquad HBH = B^{-1}. \tag{55}$$

Consequently B and H generate the translation–reflection relation

$$\langle B, H \mid H^2 = I,\ HBH = B^{-1} \rangle \cong D_\infty, \tag{56}$$

because B has infinite order.

Proof. The first identity is equation *(14)*. Direct integer multiplication gives $GBG = 5B^{-1}$, and division by 5 yields the third identity because $H = G/\sqrt{5}$. The eigenvalue $\Phi^2 > 1$ gives infinite order of B. □

Proposition XII.3 (Reader fibres and historical order)**.** *For a fixed local input e, a fibre*

$$\big(\rho \circ U(\,\cdot\,, e)\big)^{-1}(r) \tag{57}$$

with more than one state identifies distinct predecessor states at the stated reader resolution. The ledger itself retains order exactly: the map

$$\operatorname{snoc} : Y^* \times Y \longrightarrow Y^* \setminus \{()\} \tag{58}$$

is bijective, with inverse given by the prefix and final element. Thus for

$$L_0 = (), \qquad L_{n+1} = \operatorname{snoc}(L_n, r_{n+1}), \tag{59}$$

each L_m is a prefix of L_n whenever $m \le n$.

Proof. Equation *(57)* is the definition of the reader fibre. Every nonempty finite sequence has a unique final element and unique preceding prefix, which proves *(58)* and the prefix relation in *(59)*. □

The contact–record operator resolves three roles that often appear together in an observed process. B supplies the reversible geometric carrier; ρ supplies the stated accessible resolution; and L supplies the historical order of retained outcomes. A reader fibre records the precise place where a coarse-grained outcome has several compatible predecessor states. The ordered ledger preserves the sequence through which that outcome became available.

A physical arrow-of-time realization begins with a declared system and environment, a physical interaction U, a measurement or coarse-graining ρ, an outcome record, a calibration connecting s, n, and τ, and the relevant energy and entropy balance *(51)*. The finite result of this section is the exact contact–record architecture and its executable toy witness; those stated physical ingredients carry it into a measurable process [8, 9].

XIII. CONCLUSION

The scalar-symmetry theorem gives a complete finite result:

$$G = [Q, \Omega], \qquad G^2 = 5I, \qquad L_s^{\mathsf{T}} G L_s = G, \qquad H L_s H = L_{-s}, \qquad L_{2\log\Phi} = Q^2. \quad (60)$$

The same real 2×2 carrier has a discrete variational realization, null coordinates, a signature-$(3,1)$ extension, and a faithful representation of $U(1)$ phase inversion. The cross-scale spine of Section II carries this finite relation into the native equations of quantum state and readout, atomic composition, energetic flow, living regulation, and computational receipt. The Standard Model, quantum gravity, and thermodynamic sectors retain their gauge, state-space, field-equation, and entropy structures. Equations *(23)*, *(33)–(35)*, *(36)–(47)*, and *(48)–(51)* state the field-theoretic requirements in the same record as the proved scalar relation. Equations *(52)–(59)* then fix the contact–record relation: B carries reversible transport, ρ states the reader resolution, and L retains historical order. The physical bridge is completed field by field through the stated interaction, calibration, observable, and balance law.

XIV. CODA: REALITY AT HUMAN SCALE

Ordinary reality becomes readable in a simple sequence: something happens; a boundary is crossed; a signal registers a difference; a record makes a later comparison available. A clock, a camera, a laboratory spectrum, and a cell membrane each carry this relation in a material form. Coordinates, units, calibration, and source fix its resolution. This is the concrete order of address, carrier, boundary, contact, and receipt.

Geometry and light. A sunrise, a moving vehicle, and a clock face are local events in a Lorentzian geometry. Proper time orders a timelike path; light follows a null path; and an observer resolves an arriving null direction into measured energy and spatial direction by *(20)–(22)*. The 2×2 root supplies a specified Lorentzian two-plane inside that standard geometry, whose field equation is *(23)*.

Matter and chemistry. Atoms and molecules carry spectra, bound states, and interaction energies. Chemistry tracks how matter and free energy move through a stated process. For an ion of charge z moving from phase 1 to phase 2, the electrochemical difference in a declared sign convention is

$$\Delta\tilde{\mu} = RT \log \frac{a_2}{a_1} + zF(\psi_2 - \psi_1), \tag{53}$$

where a_i are activities and ψ_i are electric potentials [10]. A battery, an electrolyte, and a cell membrane all make such a difference materially available.

Living regulation. A living cell maintains a boundary, exchanges matter and energy, and couples chemical gradients to work. In oxidative phosphorylation, redox transport supports a proton-motive gradient; ATP synthase couples that gradient to ATP production through rotary catalysis [11, 12]. A viable state $x(t)$ remains within a declared range under its dynamics and feedback. This gives a physical instance of a boundary, a directed operation, a maintained difference, and a continuing record.

Experience and record. To say that a person crosses a room, a bird changes perch, or a clock hand advances is to retain an earlier state while reading a later one. The finite map in *(5)–(6)* has the same formal order: the phase returns, $Q^2 z$ advances, q_G remains fixed, and the occurrence carries an address. Section XII separates the moving state, the stated reader, and the retained sequence: B carries the state, ρ names the visible resolution, and L retains the order in which outcomes become available. The physical systems retain their native dynamics and evidence; the exact root supplies a finite grammar for keeping their common formal relations readable.

The ordinary reader therefore begins with what can be held together: a coordinate, a condition, an operation, a measurable difference, and a record. The relation is complete when a later reader can reproduce the comparison at the stated resolution.

A. THE QUADRATIC INTEGER FORM

For $x = a + b\Phi \in \mathbb{Z}[\Phi]$, define

$$N_{\text{alg}}(x) = a^2 + ab - b^2, \qquad z = \begin{pmatrix} b \\ a \end{pmatrix}. \tag{A1}$$

Then

$$z^{\mathsf{T}} G z = 2N_{\text{alg}}(x). \tag{A2}$$

Multiplication by Φ sends (a, b) to $(b, a + b)$, and consequently

$$N_{\text{alg}}(\Phi x) = -N_{\text{alg}}(x), \qquad N_{\text{alg}}(\Phi^2 x) = N_{\text{alg}}(x). \tag{A3}$$

Equations *(A2)–(A3)* are the integer form of the sign reversal under Q and the isometry under Q^2.

B. THE K_8 EXTERIOR-SQUARE CARRIER

Write the vertex set of K_8 as $\{\infty\} \cup \mathbb{Z}_7$. For $r \in \mathbb{Z}_7$, let

$$M_r = \{\{\infty, r\}\} \cup \big\{\{r + j, r - j\} : j = 1, 2, 3\big\}. \tag{A4}$$

Each M_r has four edges, and the seven matchings partition $E(K_8)$: an edge incident to ∞ determines r, while an edge $\{a, b\} \subset \mathbb{Z}_7$ occurs in the matching with

$$r = 4(a + b) \pmod 7. \tag{A5}$$

Thus every edge occurs exactly once.

Let $(t_1, \ldots, t_8)$ be an orthonormal Gell–Mann basis of $\mathfrak{su}(3)$ and write

$$[t_a, t_b] = \sum_{c=1}^{8} f_{abc} t_c. \tag{A6}$$

The adjoint matrices $(\operatorname{ad} t_a)_{bc} = -f_{abc}$ are skew and bracket-preserving, giving the exact representation

$$\operatorname{ad} : \mathfrak{su}(3) \longrightarrow \mathfrak{so}(8). \tag{A7}$$

The induced representation on $\Lambda^2\mathbb{R}^8$ has the exact spectral split

$$\Lambda^2(8) = 8 \oplus W_{20}, \qquad 28 = 8 + 20. \tag{A8}$$

In the compact skew convention, its quadratic Casimir has spectrum $\{-3, -6\}$ and satisfies

$$(\mathcal{C}_2 + 3I)(\mathcal{C}_2 + 6I) = 0, \qquad \operatorname{tr} \mathcal{C}_2 = -144. \tag{A9}$$

Thus the K_8 edge basis gives

$$\mathbb{R}^{E(K_8)} \xrightarrow{\cong} \Lambda^2\mathfrak{su}(3), \qquad e_{ij} \longmapsto t_i \wedge t_j, \tag{A10}$$

while $\mathfrak{so}(8)$ is its ambient skew-endomorphism presentation. The matching ledger is a separately proved finite address space. Its root comparison has continuous transport L_s preserving q_G and mirror relation $HL_sH = L_{-s}$; an explicit map from the ledger into

the mirror-graded algebra is the next linking construction. The square phase cell is the periodic quotient of the infinite dihedral translation–reflection reader:

$$D_\infty/\langle r^4\rangle \cong D_4, \qquad D_\infty = \langle r, s \mid s^2 = 1,\ srs = r^{-1}\rangle. \tag{A11}$$

Finite arithmetic. The Fibonacci and Lucas sequences give

$$L_5^2 + 4^2 = 11^2 + 4^2 = 137 = 1 + 4F_9. \tag{A12}$$

In the finite field $\mathbb{F}_{137}$,

$$4^{17} \equiv -1 \pmod{137}, \qquad \operatorname{ord}_{137}(4) = 34, \qquad \mathbb{F}_{137}^\times/\langle 4\rangle \cong C_4. \tag{A13}$$

These are exact finite-field identities.

C. EXECUTABLE VERIFICATION

The source package carries deterministic checks for the finite constructions. Each command, hash, result, and stated boundary is retained in the executable receipt accompanying this paper.

Carrier	Executable witness	Verified result
Root and typed gate	Root translator and mirror audit	Exact root identities, six typed `can|fold` paths, and nine declared finite gates
Physical-interface witnesses	Physical-reality verifier	Ten exact finite witnesses for the declared formulas and interfaces
Scalar and K_8 carrier	Scalar-symmetry audit	Mirror grading, all 64 basis super-Jacobi triples, scalar oscillator, matching ledger, and $\mathfrak{su}(3)$ exterior square
Contact and record	Contact–record audit	B transport, mirror reversal, reader-fibre collision, and ordered `snoc` receipt witness
Compile-time carrier	C++ phase and gate assertions	C++17/C++20 syntax and declared compile-time assertions

RECEIPT. The executable receipt records the command, SHA-256 identity, result, and boundary for each artifact.

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